Exam III-C
Key

1. a) $f'(x) = x\, 3(4x+2)^2\, 4 + (4x+2)^3 = (4x+2)^2(16x+2)$

b) $g'(x) = \dfrac{1}{2}\dfrac{1}{\sqrt{\dfrac{x^2+4}{x^2+1}}}\dfrac{(x^2+1)2x - (x^2+4)2x}{(x^2+1)^2} = \sqrt{\dfrac{x^2+1}{x^2+4}}\,\dfrac{-3x}{(x^2+1)^2}$

c) $h'(x) = \dfrac{-1}{\sqrt{1-(2x-3)^2}} \cdot 2 = \dfrac{-2}{\sqrt{1-(2x-3)^2}}$

d) $k'(x) = -\sin(\cos x)(-\sin x) = \sin(\cos x)\sin x$

e) $\ln j(x) = x \ln(2x-1)$

$$\frac{j'(x)}{j(x)} = x\frac{1}{2x-1}2 + \ln(2x-1)$$

$$j'(x) = (2x-1)^x\left(\frac{2x}{2x-1} + \ln(2x-1)\right)$$

2. a) $2x + 3(xy' + y) - 4y^3y' = 0$

$$(3x - 4y^3)y' = -2x - 3y$$

$$y' = \frac{-2x-3y}{3x-4y^3}$$

b) $3x^2 + y' = 2x - 2yy'$

$$(1+2y)y' = 2x - 3x^2$$

$$y' = \frac{2x-3x^2}{1+2y}$$

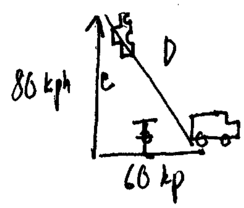
3. $$8x + 2yy' = 5(xy' + y)$$

$$(2y - 5x)y' = 5y - 8x$$

$$y' = \frac{5y - 8x}{2y - 5x} \qquad y'\Big|_{\substack{x=2\\y=3}} = \frac{15-16}{6-10} = \frac{1}{4}$$

eq. tan. line $\quad y - 3 = \frac{1}{4}(x-2)$

4.



$$c^2 + T^2 = D^2$$

$$2c\frac{dc}{dt} + 2T\frac{dT}{dt} = 2D\frac{dD}{dt}$$

$\frac{dc}{dt} = 80$

$\frac{dT}{dt} = 60$

$\frac{dD}{dt} = ?$

$$\frac{dD}{dt} = \frac{c\frac{dc}{dt} + T\frac{dT}{dt}}{D}$$

at $t = 1.5$

$c = 120, \quad T = 90, \quad D = 150$

$$\frac{dD}{dt}\Big|_{t=1.5} = \frac{120(80) + 90(60)}{150} = 100 \text{ kph}$$

5. $$\sqrt[3]{27.2} \approx \sqrt[3]{27} + dy = 3 + \frac{1}{3}\frac{1}{9}(0.2) = 3.0\overline{074}$$

where $\begin{cases} dy = f'(x)\,dx \\ f'(x) = \frac{1}{3}x^{-2/3} \\ x = 27 \\ dx = 0.2 \end{cases}$

6. $f'(x) = 15x^4 - 24x^2$

$15x^4 - 24x^2 = 3x^2(5x^2 - 8) = 0 \Rightarrow x = 0,\ x = \pm\sqrt{\frac{8}{5}} = \pm 1.2649$

x	$f(x)$		
-1	-5		
0	-10		
1.2649	-16.476	$\leftarrow$	abs min value
2	22	$\leftarrow$	abs max value

7. $f'(x) = \sqrt{x}(4-6x) + \frac{1}{2}\frac{1}{\sqrt{x}}(4x - 3x^2)$

$$= \frac{2x(4-6x) + 4x - 3x^2}{2\sqrt{x}} = \frac{12x - 15x^2}{2\sqrt{x}}$$

$f'(x) = 0 \Rightarrow x = 0.8$

$x = 0$ $\quad f(0) = 0$

$x = 0.8$ $\quad f(0.8) = 1.1448668 \leftarrow$ abs. max value

$x = 2$ $\quad f(2) = -5.656854 \leftarrow$ abs min value