Exam III - B
Key

1. a) $f'(x) = x\,2(3x-4)3 + 1(3x-4)^2 = (3x-4)(9x-4)$

b) $g'(x) = \frac{1}{2}\frac{1}{\sqrt{\frac{1-x^2}{4+x^2}}}\frac{(4+x^2)(-2x)-(1-x^2)2x}{(4+x^2)^2} = \sqrt{\frac{4+x^2}{1-x^2}}\frac{-5x}{(4+x^2)^2}$

c) $h'(x) = \frac{1}{\sqrt{1-(2-3x)^2}}(-3) = \frac{-3}{\sqrt{1-(2-3x)^2}}$

d) $k'(x) = -\sin(\sin x)\cos x$

e) $\ln j(x) = x\ln(1-2x)$

$\frac{j'(x)}{j(x)} = x\frac{1}{1-2x}(-2) + \ln(1-2x)$

$j'(x) = (1-2x)^x\left(\frac{-2x}{1-2x} + \ln(1-2x)\right)$

2. a) $3x^2 + 3(x\,2yy' + y^2) - y' = 0$

$(6xy-1)y' = -3x^2 - 3y^2$

$y' = \frac{-3x^2-3y^2}{6xy-1}$

b) $4x^3 - 2y' = 2x + 3y^2y'$

$(-2-3y^2)y' = 2x - 4x^3$

$y' = \frac{2x-4x^3}{-2-3y^2}$

3. $2x + 3y^2 y' = 2(xy' + y)$

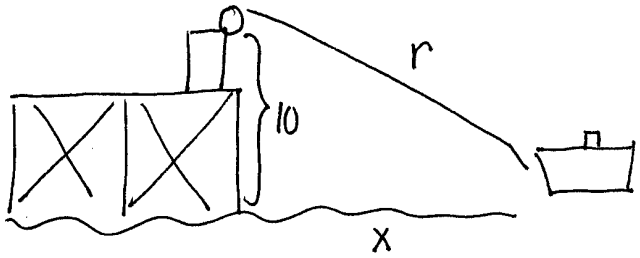
$(3y^2 - 2x)y' = 2y - 2x$

$y' = \dfrac{2y - 2x}{3y^2 - 2x}$

$y'\Big|_{\substack{x=3 \\ y=2}} = \dfrac{4-6}{12-6} = -\dfrac{1}{3}$

eq. tan. line: $y - 2 = -\frac{1}{3}(x-3)$

4.



$\dfrac{dr}{dt} = -6$ $\quad x = 24 \Rightarrow r = 26$

$\dfrac{dx}{dt} = ?$

$x^2 + 10^2 = r^2$

$2x\dfrac{dx}{dt} = 2r\dfrac{dr}{dt}$

$\dfrac{dx}{dt} = \dfrac{r}{x}\dfrac{dr}{dt}$

$\dfrac{dx}{dt}\Big|_{x=24} = \dfrac{26}{24}(-6) = -6.5$ ft/m

5. $\sqrt[4]{16.2} \approx \sqrt[4]{16} + dy = 2 + \frac{1}{4}\frac{1}{8}(0.2) = 2.00625$

where $\begin{cases} dy = f'(x)\,dx \\ f'(x) = \frac{1}{4}x^{-3/4} \\ x = 16 \\ dx = 0.2 \end{cases}$

6. $f'(x) = 20x^4 - 36x^2$

$20x^4 - 36x^2 = 4x^2(5x^2 - 9) = 0 \Rightarrow x = 0,\ x = \pm\sqrt{\frac{9}{5}} = \pm 1.34$

x	$f(x)$	
-1	10	
0	2	
1.34	-9.59	← abs. min value
2	34	← abs. max. value

7. $f'(x) = \sqrt{x}(4x-3) + \frac{1}{2\sqrt{x}}(2x^2 - 3x)$

$= \frac{2x(4x-3) + 2x^2 - 3x}{2\sqrt{x}} = \frac{10x^2 - 9x}{2\sqrt{x}}$

$f'(x) = 0 \Rightarrow x = 0.9$

$x = 0$	$f(0) = 0$	
$x = 0.9$	$f(0.9) = -1.0258$	← abs. min value
$x = 2$	$f(2) = 2.8284$	← abs. max value