Exam III-A
Key

1. a) $f'(x) = x\,2(2-3x)(-3) + (2-3x)^2 = (2-3x)(2-9x)$

b) $g'(x) = \frac{1}{2}\frac{1}{\sqrt{\frac{x^2+1}{x^2-1}}}\frac{(x^2-1)2x-(x^2+1)2x}{(x^2-1)^2} = \sqrt{\frac{x^2-1}{x^2+1}}\,\frac{-2x}{(x^2-1)^2}$

c) $h'(x) = \frac{-1}{\sqrt{1-(4x+3)^2}}\cdot 4 = \frac{-4}{\sqrt{1-(4x+3)^2}}$

d) $k'(x) = \cos(\cos x)(-\sin x)$

e) $\ln j(x) = (1-2x)\ln x$

$\frac{j'(x)}{j(x)} = (1-2x)\frac{1}{x} + (-2)\ln x$

$j'(x) = x^{1-2x}\left(\frac{1-2x}{x} - 2\ln x\right)$

2. a) $2x + 3(xy'+y) - 3y^2y' = 0$

$(3x-3y^2)y' = -2x-3y$

$y' = \frac{-2x-3y}{3x-3y^2}$

b) $2x + y' = 3x^2 + 2y\,y'$

$(1-2y)y' = 3x^2-2x$

$y' = \frac{3x^2-2x}{1-2y}$

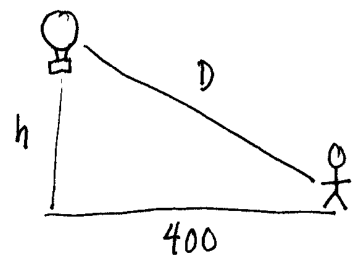
3. $3x^2 + 2yy' = 4(xy' + y)$

$(2y - 4x)y' = 4y - 3x^2$

$y' = \dfrac{4y - 3x^2}{2y - 4x}$ $\qquad y'\Big|_{\substack{x=2 \\ y=3}} = \dfrac{12-12}{6-8} = 0$

eq. tan. line $\quad y - 3 = 0(x-2)$

4.



$\dfrac{dh}{dt} = 3.5$ $\qquad h = 300 \Rightarrow D = 500$

$\dfrac{dD}{dt} = ?$

$h^2 + 400^2 = D^2$

$2h\dfrac{dh}{dt} = 2D\dfrac{dD}{dt}$ $\qquad \dfrac{dD}{dt} = \dfrac{h}{D}\dfrac{dh}{dt}$

$\dfrac{dD}{dt}\Big|_{h=300} = \dfrac{300}{500}(3.5) = 2.1\ g/s$

5. $\sqrt[3]{7.9} \approx \sqrt[3]{8} + dy = 2 + \frac{1}{3}\cdot\frac{1}{4}(-0.1) = 2 - 0.0.08\bar{3} = 1.991\bar{6}$

$dy = f'(x)\,dx$

where $\begin{cases} f'(x) = \frac{1}{3}x^{-2/3} \\ dx = -0.1 \\ x = 8 \end{cases}$

6. $f'(x) = 15x^4 - 36x^2$

$15x^4 - 36x^2 = 3x^2(5x^2 - 12) = 0 \Rightarrow x = 0, \quad x = \pm\sqrt{\frac{12}{5}} = \pm 1.55$

x	$f(x)$	
-1	$+7$	← abs. max value
0	-2	
1.55	-19.85	← abs min value
2	-2	

7. $f'(x) = \sqrt{x}\,(8x-5) + \frac{1}{2}\frac{1}{\sqrt{x}}(4x^2 - 5x)$

$$= \frac{2x(8x-5) + (4x^2 - 5x)}{2\sqrt{x}} = \frac{20x^2 - 15x}{2\sqrt{x}}$$

$f'(x) = 0 \Rightarrow x = .75$

$x = 0$	$f(0) = 0$	
$x = .75$	$f(.75) = -1.299$	← abs. min value
$x = 2$	$f(2) = 8.485$	← abs. max value