Exam II-C Key

1. $f(2) = 1 \quad \lim_{x\to 2} ax - 5 = a2 - 5 = 1 \Rightarrow a = 3$

$f(5) = 25 + 5b \quad \lim_{x\to 5} 2x - 3 = 7 \quad 25 + 5b = 7 \Rightarrow b = \frac{-18}{5}$

2. $5 \log_4 16 - 7 \log_3 9 = 5 \log_4 4^2 - 7 \log_3 3^2 = 5(2) - 7(2) = -4$

3. $4^{x^2+x} = 16 = 4^2 \Rightarrow x^2 + x = 2$

$x^2 + x - 2 = 0 \Rightarrow (x+2)(x-1) = 0$

$x = -2$ or $x = 1$

4. $\log_{12} x + \log_{12}(x+1) = 1 \Rightarrow \log_{12} x(x+1) = 1$

$x^2 + x = 12 \Rightarrow x^2 + x - 12 = 0$

$(x+4)(x-3) = 0 \Rightarrow$ ($x = -4$ extraneous) or $x = 3$

$x = 3$

5. $f'(x) = \lim_{h\to 0} \frac{f(x+h) - f(x)}{h} = \lim_{h\to 0} \frac{(x+h)^2 - 6(x+h) - (x^2 - 6x)}{h}$

$= \lim_{h\to 0} \frac{x^2 + 2xh + h^2 - 6x - 6h - x^2 + 6x}{h}$

$= \lim_{h\to 0} \frac{2xh + h^2 - 6h}{h} = \lim_{h\to 0} \frac{h(2x + h - 6)}{h} = 2x - 6$

6. a) $f'(x) = -4\frac{1}{2\sqrt{x}} + 3\left(-5x^{-6}\right) = \frac{-2}{\sqrt{x}} - \frac{15}{x^6}$

b) $g(x) = 2x - 4x^{-1} - 7x^{-2} + 5x^{-4}$

$g'(x) = 2 + \frac{4}{x^2} + \frac{14}{x^3} - \frac{20}{x^5}$

c) $h'(x) = \frac{(x^2+2x-4)(4x-3) - (2x^2-3x)(2x+2)}{(x^2+2x-4)^2}$

$= \frac{4x^3 + 8x^2 - 16x - 3x^2 - 6x + 12 - 4x^3 - 4x^2 + 6x^2 + 6x}{(x^2+2x-4)^2}$

$= \frac{7x^2 - 16x + 12}{(x^2+2x-4)^2}$

d) $k'(x) = e^x(-\sin x - \cos x) + e^x(\cos x - \sin x)$

$= e^x(-2\sin x)$

7. $f'(x) = x^3e^x + 3x^2e^x = (x^3+3x^2)e^x$

$f''(x) = (x^3+3x^2)e^x + (3x^2+6x)e^x = (x^3+6x^2+6x)e^x$

8. at $x = 12$ we have $y = \sqrt[3]{8} = 2$

$f'(x) = \frac{1}{3}\frac{1}{(x-4)^{2/3}}$ $\quad f'(12) = \frac{1}{3\cdot 4} = \frac{1}{12}$

$y - 2 = \frac{1}{12}(x-12)$

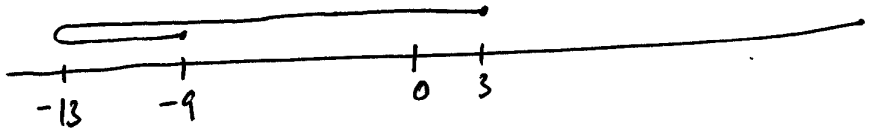
9. a) $v(t) = 8t - 16$

b) $a(t) = 8$

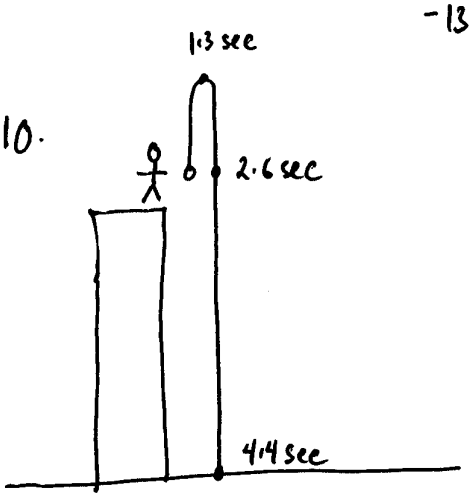
c) $v(t) = 0 \Rightarrow 8t - 16 = 0 \Rightarrow t = 2$

$s(1) = -9$
$s(2) = -13$
$s(4) = 3$

total distance $= 4 + 16 = 20$



10.



a) 41.6 ft/sec

b) 126.72 ft

c) 153.76 ft

d) -99.2 ft/sec

$s(t) = s_0 + v_0 t - 16t^2$

$v(t) = v_0 - 32t$

$v(1.3) = 0 \Rightarrow v_0 - 32(1.3) = 0 \Rightarrow v_0 = 41.6$

$s(t) = s_0 + 41.6t - 16t^2$

$s(4.4) = 0 \Rightarrow s_0 + 41.6(4.4) - 16(4.4)^2 = 0$

$\Rightarrow s_0 = 126.72$

$s(t) = 126.72 + 41.6t - 16t^2$

$s(1.3) = 126.72 + 41.6(1.3) - 16(1.3)^2$

$= 153.76$

$v(t) = 41.6 - 32t$

$v(4.4) = 41.6 - 32(4.4) = -99.2$