Exam II-B Key

1. $f(2) = 6 \qquad \lim_{x\to 2} ax-3 = a2-3 = 6 \Rightarrow a = \frac{9}{2}$

$f(5) = 25-5b \qquad \lim_{x\to 5} x+4 = 9 \qquad 25-5b = 9 \Rightarrow b = \frac{16}{5}$

2. $2\log_3 27 - 3\log_2 16 = 2\log_3 3^3 - 3\log_2 2^4 = 2(3) - 3(4) = -6$

3. $3^{x^2-x} = 9 = 3^2 \Rightarrow x^2-x = 2$

$x^2-x-2=0 \Rightarrow (x-2)(x+1)=0$

$x = 2$ or $x = -1$

4. $\log_3 x + \log_3 (x+2) = 1 \Rightarrow \log_3 x(x+2) = 1$

$x^2+2x = 3 \Rightarrow x^2+2x-3 = 0$

$(x+3)(x-1) = 0 \Longrightarrow$ $x = -3$ ex-traneous or $x = 1$

5. $f'(x) = \lim_{h\to 0} \frac{f(x+h) - f(x)}{h} = \lim_{h\to 0} \frac{2(x+h)^2 + x+h - (2x^2+x)}{h}$

$= \lim_{h\to 0} \frac{\cancel{2x^2} + 4xh + 2h^2 + \cancel{x} + h - \cancel{2x^2} - \cancel{x}}{h}$

$= \lim_{h\to 0} \frac{4xh + 2h^2 + h}{h} = \lim_{h\to 0} \frac{\cancel{h}(4x + 2h + 1)}{\cancel{h}} = 4x+1$

6. a) $f'(x) = 6\frac{1}{2\sqrt{x}} + 2\left(-3x^{-4}\right) = \frac{3}{\sqrt{x}} - \frac{6}{x^4}$

b) $g(x) = 2x^2 + 5x^{-1} - 7x^{-2} + 2x^{-3}$

$g'(x) = 4x - \frac{5}{x^2} + \frac{14}{x^3} - \frac{6}{x^4}$

c) $h(x) = \frac{2x^2+3x}{-x^2+4}$

$h'(x) = \frac{(-x^2+4)(4x+3) - (2x^2+3x)(-2x)}{(-x^2+4)^2}$

$= \frac{-4x^3 - 3x^2 + 16x + 12 + 4x^3 + 6x^2}{(-x^2+4)^2}$

$= \frac{3x^2+16x+12}{(-x^2+4)^2}$

d) $k'(x) = e^x(-\sin x + \cos 2x) + e^x(\cos x + \sin x) = 2e^x \cos x$

7. $f'(x) = x^2 e^{-x}(-1) + 2x e^{-x} = e^{-x}(2x - x^2) = (-x^2+2x)e^{-x}$

$f''(x) = (-x^2+2x)e^{-x}(-1) + (-2x+2)e^{-x}$

$= (x^2 - 4x + 2)e^{-x}$

8. at $x = 4$ we have $y = \sqrt[3]{8} = 2$

$f'(x) = \frac{1}{3}\frac{1}{(x+4)^{2/3}}$ $\quad f'(4) = \frac{1}{3 \cdot 4} = \frac{1}{12}$

$y - 2 = \frac{1}{12}(x-4)$

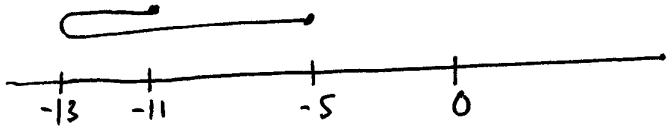
9. a) $v(t) = 4t - 12$

b) $a(t) = 4$

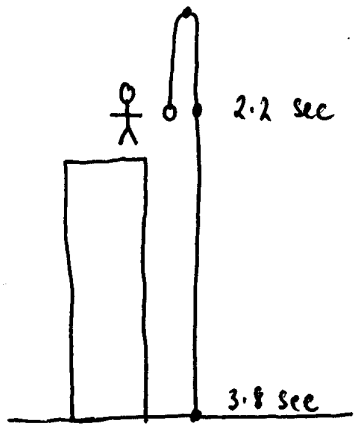
c) $v(t) = 0 \Rightarrow 4t - 12 = 0 \Rightarrow t = 3$

$s(1) = -5$
$s(3) = -13$
$s(4) = -11$

total distance $= 8 + 2 = 10$



10.



a) 35.2 ft/sec

b) 97.28 feet

c) 116.64 feet

d) −86.4 ft/sec

$s(t) = s_0 + v_0 t - 16t^2$

$v(t) = v_0 - 32t$

$v(1.1) = 0 \Rightarrow v_0 - 32(1.1) = 0 \Rightarrow v_0 = 35.2$

$s(t) = s_0 + 35.2t - 16t^2$

$s(3.8) = 0 \Rightarrow s_0 + 35.2(3.8) - 16(3.8)^2 = 0$

$\Rightarrow s_0 = 97.28$

$s(t) = 97.28 + 35.2t - 16t^2$

$s(1.1) = 97.28 + 35.2(1.1) - 16(1.1)^2$

$= 116.64$

$v(t) = 35.2 - 32t$

$v(3.8) = 35.2 - 32(3.8) = -86.4$