Exam II-A Key

1. $f(2) = -2 \qquad \lim_{x\to 2} ax+3 = a2+3 = -2 \Rightarrow a = -\frac{5}{2}$

$f(5) = 25-5b \qquad \lim_{x\to 5} x-4 = 1 \qquad 25-5b = 1 \Rightarrow b = \frac{24}{5}$

2. $5\log_2 16 - 6\log_3 9 = 5\log_2 2^4 - 6\log_3 3^2 = 5(4) - 6(2) = 8$

3. $2^{x^2-2x} = 8 = 2^3 \Rightarrow x^2-2x = 3$

$x^2-2x-3 = 0 \Rightarrow (x-3)(x+1) = 0$

$x = 3$ or $x = -1$

4. $\log_6 x + \log_6 (x+1) = 1 \Rightarrow \log_6 x(x+1) = 1$

$x^2+x = 6 \Rightarrow x^2+x-6 = 0$

$(x+3)(x-2) = 0 \Rightarrow$ $x = -3$ extraneous or $x = 2$

5. $f'(x) = \lim_{h\to 0} \frac{f(x+h)-f(x)}{h} = \lim_{h\to 0} \frac{(x+h)^2-4(x+h)-(x^2-4x)}{h}$

$= \lim_{h\to 0} \frac{\cancel{x^2}+2xh+h^2-\cancel{4x}-4h-\cancel{-x^2}+\cancel{4x}}{h}$

$= \lim_{h\to 0} \frac{2xh+h^2-4h}{h} = \lim_{h\to 0} \frac{\cancel{h}(2x+h-4)}{\cancel{h}} = 2x-4$

6. a) $f'(x) = 4\frac{1}{2\sqrt{x}} - 3\left(-4x^{-5}\right) = \frac{2}{\sqrt{x}} + \frac{12}{x^5}$

b) $g(x) = 2x - 4x^{-1} - 7x^{-2} + x^{-4}$

$g'(x) = 2 + \frac{4}{x^2} + \frac{14}{x^3} - \frac{4}{x^5}$

c) $h'(x) = \frac{(x^2+2x+4)(4x-3) - (2x^2-3x)(2x+2)}{(x^2+2x+4)^2}$

$= \frac{4x^3 + 8x^2 + 16x - 3x^2 - 6x - 12 - 4x^3 - 4x^2 + 6x^2 + 6x}{(x^2+2x+4)^2}$

$= \frac{7x^2 + 16x - 12}{(x^2+2x+4)^2}$

d) $k'(x) = e^x(\cos x + \sin x) + e^x(\sin x - \cos x)$

$= 2e^x \sin x$

7. $f'(x) = x^2e^x + 2xe^x = (x^2+2x)e^x$

$f''(x) = (x^2+2x)e^x + (2x+2)e^x = (x^2+4x+2)e^x$

8. at $x=6$ we have $y = \sqrt[3]{8} = 2$

$f'(x) = \frac{1}{3}\frac{1}{(x+2)^{2/3}}$ $\quad f'(6) = \frac{1}{3\cdot 4} = \frac{1}{12}$

$y - 2 = \frac{1}{12}(x-6)$

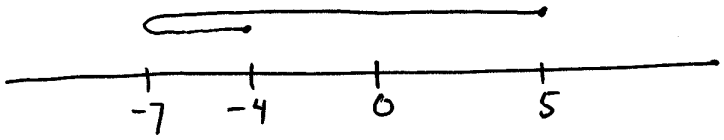
9. a) $v(t) = 6t - 12$

b) $a(t) = 6$

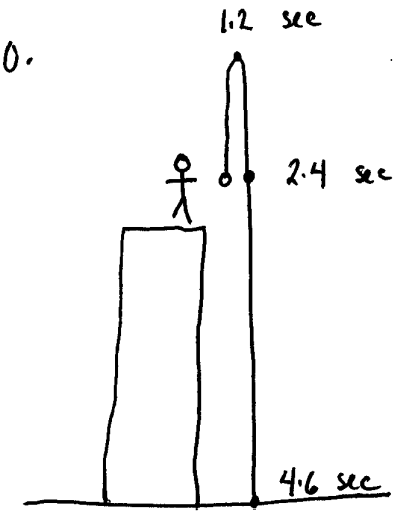
c) $v(t) = 0 \Rightarrow 6t - 12 = 0 \Rightarrow t = 2$

$s(1) = -4$
$s(2) = -7$
$s(4) = 5$

total distance $= 3 + 12 = 15$



10.



$s(t) = s_0 + v_0 t - 16t^2$

$v(t) = v_0 - 32t$

$v(1.2) = 0 \Rightarrow v_0 - 32(1.2) = 0 \Rightarrow v_0 = 38.4$

$s(t) = s_0 + 38.4t - 16t^2$

$s(4.6) = 0 \Rightarrow s_0 + 38.4(4.6) - 16(4.6)^2 = 0$

$\Rightarrow s_0 = 161.92$

$s(t) = 161.92 + 38.4t - 16t^2$

$s(1.2) = 161.92 + 38.4(1.2) - 16(1.2)^2$

$= 184.96$

$v(t) = 38.4 - 32t$

$v(4.6) = 38.4 - 32(4.6) = -108.8$

a) 38.4 ft/sec

b) 161.92 ft.

c) 184.96 ft

d) -108.8 ft/sec