Inverse Invariant Theory and Steenrod Operations

Addenda, Corrections, and Comments

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I will leave out obvious typos.

PAGE 45 : Theorem 2.4.1 can't be true: The extension $\mathbb{F}(V)/\mathbf{FF}(D^*(V))$ is a counterexample. The correct statement of Theorem 2.4.1 is the following:

THEOREM 2.4.1' : *Let $\mathbb{K}^* \hookrightarrow \mathbb{L}^*$ be a field extension over the Steenrod algebra. If*

$$\dim_{\mathbb{K}^*}(\Delta(\mathbb{K}^*)) = \dim_{\mathbb{L}^*}(\Delta(\mathbb{L}^*)) = m < \infty$$

then either $\mathbb{K}^* \hookrightarrow \mathbb{L}^*$ is separable or $\mathbb{L}^*$ is not P^*-inseparably closed.

The proof of Theorem 2.4.1 shows precisely this. However, some steps need some elaboration:

PROOF: If $\mathbb{K}^* \hookrightarrow \mathbb{L}^*$ is separable, then nothing is to be shown. So assume that $\mathbb{K}^* \hookrightarrow \mathbb{L}^* = \mathbb{K}^*(\mathscr{S})$ is a non-separable field extension, where $\mathscr{S}$ is a minimal generating set. Denote by $\overline{\mathbb{K}^*}$ the algebraic closure of $\mathbb{K}^*$ in $\mathbb{L}^*$. Then by construction $\mathbb{K}^* \hookrightarrow \overline{\mathbb{K}^*}$ is purely inseparable, and $\overline{\mathbb{K}^*} \hookrightarrow \mathbb{L}^*$ is transcendental.

The field $\overline{\mathbb{K}^*}$ inherits an action of the Steenrod algebra from $\mathbb{L}^*$ as we see next. For any $k \in \overline{\mathbb{K}^*}$ and any $i \in \mathbb{N}_0$ we have that $\mathscr{P}^i(k) \in \mathbb{L}^*$. Moreover $k^{p^e} \in \mathbb{K}^*$ for some $e \in \mathbb{N}_0$. Since

$$(\mathscr{P}^i(k))^{p^e} = \mathscr{P}^{ip^e}(k^{p^e}) \in \mathbb{K}^*$$

is purely inseparable over $\mathbb{K}^*$, also $\mathscr{P}^i(k) \in \overline{\mathbb{K}^*}$ for all $i \in \mathbb{N}_0$.

Thus we can consider the cases where $\mathbb{K}^* \hookrightarrow \mathbb{L}^*$ is purely inseparable, resp. purely transcendental, separately. We start with some preliminary results.

Denote by ${\mathbb{K}^*}_c$, resp. ${\mathbb{L}^*}_c$, the subfield of $\Delta(\mathbb{K}^*)$-constants, resp. $\Delta(\mathbb{L}^*)$-constants. Then by Theorem 19 on page 186 of [17], the field extensions

$${\mathbb{K}^*}_c \hookrightarrow \mathbb{K}^*, \text{ and, } {\mathbb{L}^*}_c \hookrightarrow \mathbb{L}^*$$

both are purely inseparable of degree p^m. We have the following diagram

$$\begin{array}{ccccc} & & {\mathbb{K}^*}_c & \underset{p^m}{\hookrightarrow} & \mathbb{K}^* \\ & \swarrow & \downarrow & & \downarrow \\ {\mathbb{L}^*}_c & \underset{p^{m_1}}{\hookrightarrow} & {\mathbb{K}^*}_c(\mathscr{S}) & \underset{p^{m_2}}{\hookrightarrow} & \mathbb{L}^* = \mathbb{K}^*(\mathscr{S}), \end{array}$$

where the labels at the extensions denote the degree. By construction $m_1 + m_2 = m$.

CASE : $\mathbb{K}^* \hookrightarrow \mathbb{L}^*$ is purely inseparable

The extension ${\mathbb{K}^*}_c \hookrightarrow {\mathbb{K}^*}_c(S)$ has the same degree as the extension $\mathbb{K}^* \hookrightarrow \mathbb{L}^*$. To see this note first that $|\mathbb{L}^* : \mathbb{K}^*| \leq |{\mathbb{K}^*}_c(S) : {\mathbb{K}^*}_c|$ by construction. Furthermore, for any $k \in {\mathbb{K}^*}_c(S) \subseteq \mathbb{L}^*$ we have a minimal e such that $k^{p^e} \in \mathbb{K}^*$. If $|\mathbb{L}^* : \mathbb{K}^*| < |{\mathbb{K}^*}_c(S) : {\mathbb{K}^*}_c|$ then there were an element $k \in {\mathbb{K}^*}_c$ such that $k^{p^e} \in \mathbb{K}^*$ but $k^{p^e} \notin {\mathbb{K}^*}_c$. Since pth powers are constant, this can't happen. Therefore

$${\mathbb{K}^*}_c(\mathscr{S}) \hookrightarrow \mathbb{K}^*(\mathscr{S}) = \mathbb{L}^*$$

is purely inseparable of degree p^m. Hence $m = m_2$, $m_1 = 0$, and so

$${\mathbb{L}^*}_c = {\mathbb{K}^*}_c(\mathscr{S}).$$

This, in turn, means that any element $s \in \mathscr{S}$ is a $\Delta(\mathbb{L}^*)$-constant and, since $\mathscr{S}$ was minimal, has no p-th root in $\mathbb{L}^*$. Therefore $\mathbb{L}^*$ is not P^*-inseparably closed.

CASE : $\mathbb{K}^* \hookrightarrow \mathbb{L}^*$ is purely transcendental

If $\mathbb{K}^* \hookrightarrow \mathbb{L}^*$ is purely transcendental its transcendence degree is equal to the cardinality of the set $\mathscr{S}$. We need to show that $\mathbb{K}^*{}_c(\mathscr{S}) = \mathbb{L}^*{}_c$. Since $\mathbb{L}^* = \mathbb{K}^*(\mathscr{S})$ is purely transcendental, we have that

$$\left|\mathbb{K}^* : \mathbb{K}^*{}_c\right| = \left|\mathbb{K}^*(\mathscr{S}) : \mathbb{K}^*{}_c(\mathscr{S})\right| = \left|\mathbb{L}^* : \mathbb{K}^*{}_c(\mathscr{S})\right|.$$

Thus $m_2 = m$ and $\mathbb{L}^*{}_c = \mathbb{K}^*{}_c(\mathscr{S})$ as desired. •

PAGE 46 : `Since the statement of Theorem 2.4.1 had to be corrected Corollary 2.4.2 needs a new proof.`

COROLLARY 2.4.2 : *Let* $\mathbb{F}(x_1, \ldots, x_n) \hookrightarrow \mathbb{L}^*$ *be a field extension of fields over the Steenrod algebra, and let*

$$\dim_{\mathbb{L}^*}(\Delta(\mathbb{L}^*)) = n.$$

$\mathbb{L}^*$ *is* P^**-inseparably closed if and only if* $\mathbb{F}(x_1, \ldots, x_n) = \mathbb{L}^*$.

PROOF: If $\mathbb{L}^*$ is not P^*-inseparably closed, it can't be equal to $\mathbb{F}(x_1, \ldots, x_n)$, because the smaller field is P^*-inseparably closed, by Corollary 2.3.4.

For the converse, assume that $\mathbb{L}^*$ is P^*-inseparably closed. By Theorem 1.2.3, the module of derivations, $\Delta(\mathbb{F}[x_1, \ldots, x_n])$, is a free module over the polynomial ring generated by n elements. Hence the vector space of derivations, $\Delta(\mathbb{F}(x_1, \ldots, x_n))$, has also dimension n. So by the preceding theorem, Theorem 2.4.1', the extension $\mathbb{F}(V) \hookrightarrow \mathbb{L}^*$ is separable. By Theorem 3.2.2 the field $\mathbb{F}(V)$ is algebraically closed, thus $\mathbb{F}(V) = \mathbb{L}^*$ as claimed. •

`Well, yes, Theorem 3.2.2 is proven after Corollary 2.4.2, but it does not depend on in.`

PAGE 54, line 11 : ... is algebraic over R, i.e., satisfies an algebraic relation with coefficients in R.

PAGE 58, line -2 : $\cdots$ generated by the products $\gamma_1^{j_1} \cdots \gamma_t^{j_t}$ for $j_i = 1, \ldots, p-1$ and $i = 1, \ldots, t$ because $\gamma_i^p = c_i \in \mathrm{H}^*$ ...

PAGE 65, line 11 : $(x_2,\ x_3, \ldots) \subsetneqq (x_1,\ x_2,\ x_3, \ldots)$

PAGE 85, line 6 : $(-1)^k \varphi^{-1}(\mathbf{d}_{k,0})\mathscr{P}^{\Delta_0} + \cdots + (-1)\varphi^{-1}(\mathbf{d}_{k,k-1})\mathscr{P}^{\Delta_{k-1}} + \mathscr{P}^{\Delta_k} = 0$

PAGE 87ff : `There are some problems in Section 5.3, because Lemma 5.3.1 is wrong. We start with some lemmata in order to make future reference easier. The first result is part of Lemma 1.4 in [1].`

LEMMA 1 : *Let* H^* *be an unstable integral domain. Then* $\mathbf{FF}(\mathrm{H}^*) = \mathbf{FF}(Un\,(\mathbf{FF}(\mathrm{H}^*)))$

PROOF: Since $\mathrm{H}^* \subseteq Un\,(\mathbf{FF}(\mathrm{H}^*))$ the inclusion "$\subseteq$" is clear. To prove the reverse inclusion, let $f \in \mathbf{FF}(Un\,(\mathbf{FF}(\mathrm{H}^*)))$. Then $f = \frac{f_1}{f_2}$ for some elements in $f_1,\ f_2 \in Un\,(\mathbf{FF}(\mathrm{H}^*)) \subseteq \mathbf{FF}(\mathrm{H}^*)$. Thus there exist elements $h_1, \ldots, h_4 \in \mathrm{H}^*$ such that

$$f = \frac{f_1}{f_2} = \frac{h_1/h_2}{h_3/h_4} = \frac{h_1 h_4}{h_2 h_3}$$

which is in $\mathbf{FF}(\mathrm{H}^*)$. •

LEMMA 2 : *Let* H^* *be an unstable integral domain. Let* $\mathbf{FF}(\mathrm{H}^*)$ *be* P^**-inseparably closed. Then also* $Un\,(\mathbf{FF}(\mathrm{H}^*))$ *is* P^**-inseparably closed.*

PROOF: Take an element $h \in \sqrt[P^*]{Un\,(\mathbf{FF}(\mathrm{H}^*))}$ such that $h^p \in Un\,(\mathbf{FF}(\mathrm{H}^*)) \subseteq \mathbf{FF}(\mathrm{H}^*)$. Since the field of fractions is by assumption P^*-inseparably closed, we know that $h \in \mathbf{FF}(\mathrm{H}^*)$. But h being the pth root of an unstable element is unstable. Thus $h \in Un\,(\mathbf{FF}(\mathrm{H}^*))$ as desired. •

The third result was used throughout the text, but it seems reasonable to phrase it explicitly, cf. Theorem 5.2.1.

LEMMA 3 : *Let* H^* *be an unstable Noetherian integral domain. Then the algebra*

$$A^* = \mathrm{H}^* < z_1, \ldots, z_m >$$

obtained from H^* *by adjoining the roots of the* Δ*-polynomial is Noetherian with the same Krull dimension as* H^*.

PROOF: Note that the Δ-length, m, of H^* is at most the Krull dimension of H^* by Corollary 1.2.2. The elements $z_1, \ldots, z_m$ are the roots of the polynomial

$$\Delta(X) = \mathbf{t}_0 X + \cdots + \mathbf{t}_{m-1} X^{q^{m-1}} + \mathbf{t}_m X^{q^m} \in \mathbf{FF}(\mathrm{H}^*)[X].$$

By Theorem 1.2.1 we have for $j = 1, \ldots, m$

$$z_j^{q^{m+1}} = \mathcal{P}^{m+1}(z_j) = a_0 \mathcal{P}^{\Delta_0}(z_j) + \cdots + a_m \mathcal{P}^{\Delta_m}(z_j) = a_0 z_j + \cdots + a_m z_j^{q^m}$$

for suitable elements $a_0, \ldots, a_m \in \mathrm{H}^*$. Thus $\mathrm{H}^* \hookrightarrow A^*$ is an integral extension. Furthermore, A^* is finitely generated as an algebra over H^*. Thus the extension $\mathrm{H}^* \hookrightarrow A^*$ is finite. Since H^* is Noetherian, so is A^*. For the same reason, the Krull dimension remains the same for both algebras. •

We correct Lemma 5.3.1:

LEMMA 5.3.1' : $Un\,(\mathbf{FF}(\mathrm{H}^*))$ *is* P^**-inseparably closed if and only if* $Un\,(\mathbf{FF}(A^*))$ *is.*

PROOF: By Lemma 1 we have $\mathbf{FF}(\mathrm{H}^*) = \mathbf{FF}(Un\,(\mathbf{FF}(\mathrm{H}^*)))$. Let $Un\,(\mathbf{FF}(\mathrm{H}^*))$ be P^*-inseparably closed. Then looking at the fields of fractions we get

$$\begin{array}{ccc} \mathbf{FF}(\mathrm{H}^*) & \underset{\text{separable}}{\hookrightarrow} & \mathbf{FF}(A^*) \\ & & {\scriptstyle P^*\text{–purely}} \downarrow {\scriptstyle \text{insep.}} \\ & & \mathbf{FF}(\sqrt[P^*]{A^*}), \end{array}$$

where the upper map is P^*-separable, because $\mathbf{FF}(A^*) = \mathbb{E}^*$ is the splitting field of the separable polynomial $\Delta(X)$, and the pure P^*-inseparability of the map downwards follows from Lemma 4.2.5. So, we find an intermediate field $\mathbb{K}^*$ between $\mathbf{FF}(\mathrm{H}^*)$ and $\mathbf{FF}(\sqrt[P^*]{A^*})$, such that

$$\mathbf{FF}(\mathrm{H}^*) \subseteq \mathbb{K}^*$$

is P^*-purely inseparable, while

$$\mathbb{K}^* \subseteq \mathbf{FF}(\sqrt[P^*]{A^*})$$

is P^*-separable, see the remarks at the end of Section 2.3. However, by Corollary 4.2.7, $\mathbf{FF}(\mathrm{H}^*)$ is P^*-inseparably closed, which means that $\mathbb{K}^* = \mathbf{FF}(\mathrm{H}^*)$ and

$$\mathbf{FF}(\mathrm{H}^*) \hookrightarrow \mathbf{FF}(\sqrt[P^*]{A^*})$$

is P^*-separable. Thus $\mathbf{FF}(\mathrm{A}^*)$ is P^*-inseparably closed, and thus so must be $Un\,(\mathbf{FF}(A^*))$ by Lemma 2.

To prove the converse assume that $Un\,(\mathbf{FF}(\mathrm{H}^*))$ is not P^*-inseparably closed. By Lemma 4.2.5 and Corollary 4.2.7 we have a commutative diagram

$$\begin{array}{ccc} \mathbf{FF}(\mathrm{H}^*) & \underset{P^*\text{–purely insep.}}{\hookrightarrow} & \mathbf{FF}(\sqrt[P^*]{\mathrm{H}^*}) \\ \downarrow & & \downarrow \\ \mathbf{FF}(A^*) & \underset{P^*\text{–purely insep.}}{\hookrightarrow} & \mathbf{FF}(\sqrt[P^*]{A^*}). \end{array}$$

Since $\mathbf{FF}(A^*) = \mathbb{E}^*$ is a splitting field of a P^*-separable polynomial, namely of $\Delta(X)$, the extension $\mathbf{FF}(A^*)/\mathbf{FF}(\mathrm{H}^*)$ is P^*-separable. Therefore $\mathbf{FF}(\sqrt[P^*]{A^*})/\mathbf{FF}(A^*)$ is a proper P^*-inseparable extension as claimed. •

As noted in Corollary 4.2.7 the field of fractions of a P^*-inseparable closed integral domain is P^*-inseparably closed. The converse, however is not true in general, cf. Example 1 in Section 7.4. This is precisely the problem with the original proof. We avoided this, by replacing the algebras H^* and A^* by the unstable parts of their respective fields of fractions. Recall from Corollary 4.2.7 that if H^* is integrally closed and its field of fractions is P^*-inseparably closed, then so is the algebra H^*. Indeed, for any algebra H^* the unstable part $Un\,(\mathbf{FF}(\mathrm{H}^*))$ coincides with the integral closure of H^*. A statement that I was not able to prove back then, and indeed its proof relies on the results of this memoir, cf. Theorem 2.4 in [1].

Lemma 5.3.2 is just fine. Note that it works also under the weaker assumption that $Un\,(\mathbf{FF}(A^*))$ is P^*-inseparably closed. The same comments apply to Theorem 5.3.3.

We come to Theorem 5.3.4. Since Lemma 5.3.1 was not correct, we need to refine the proof.

Theorem 5.3.4 : *Let H^* be an P^*-inseparably closed unstable integral domain of Δ-length m. Then H^* can be embedded into a polynomial ring with m linear generators. Moreover, the embedding is integral if and only if H^* is Noetherian.*

Proof: Since H^* is P^*-inseparably closed, so is its field of fractions by Corollary 4.2.7. Therefore by Lemma 2 the unstable part $Un\,(\mathbf{FF}(\mathrm{H}^*))$ is also P^*-inseparably closed. Thus the algebra $Un\,(\mathbf{FF}(A^*))$ is P^*-inseparably closed by Lemma 5.3.1'. So, Theorem 5.3.3 shows that the map ψ is an isomorphism, i.e.,

$$\mathrm{H}^* \hookrightarrow A^* \overset{\psi}{\cong} \mathbb{F}[z_1, \ldots, z_m]$$

is the embedding we wanted. The remainder of the proof stays as stated. •

Now, finally we need to correct Theorem 5.3.5.

Theorem 5.3.5' : *Let H^* be a Δ-finite unstable integral domain. Let A^* be obtained from H^* by adjoining all roots $z_1, \ldots, z_m$ of the Δ-polynomial of H^*. The following statements are equivalent:*

(1) $Un\,(\mathbf{FF}(\mathrm{H}^*))$ *is P^*-inseparably closed.*
(2) $Un\,(\mathbf{FF}(A^*))$ *is P^*-inseparably closed.*
(3) *The inclusion $\psi : \mathbb{F}[z_1, \ldots, z_m] \hookrightarrow A^*$ is an isomorphism.*

Moreover, if H^* is Noetherian, the above statements imply that the Δ-length m of H^* is equal to its Krull dimension.

PROOF: The equivalence of the first two statements is the contents of Lemma 5.3.1'. Statement (3) implies (2) by Lemma 5.3.2, because $Un\,(\mathbf{FF}(A^*))$ is P^*-inseparably closed if A^* is by Lemma 2. In Theorem 5.3.3 we have seen that (2) $\Rightarrow$ (3). The remainder of the proof stays as it is. •

Another note: In the footnote to Theoerm 5.3.5 a converse is announced for Corollary 6.1.4. This converse is wrong as we will see later.

PAGE 94 : Theorem 6.1.3 makes use of Lemma 5.3.1. Since this was wrong, we need to correct the proof. The proof works as stated until line 18 of page 95. We take it from there:

... Since $\sqrt[P^*]{\mathrm{H}^*}$ is P^*-inseparably closed, so is $Un\,(\mathbf{FF}(\sqrt[P^*]{\mathrm{H}^*} < z_1, \ldots, z_m >))$ by Lemma 2. We look at the fields of fractions involved

$$\begin{array}{ccccc} \mathbb{F}[z_1,\ldots,z_m] & \hookrightarrow & \mathrm{H}^* < z_1,\ldots,z_m > & \underset{\text{integral}}{\hookrightarrow} & \sqrt[P^*]{\mathrm{H}^*} < z_1,\ldots,z_m >, \\ \downarrow & & \downarrow & & \downarrow \\ \mathbb{F}(z_1,\ldots,z_m) & \hookrightarrow & \mathbf{FF}(\mathrm{H}^* < z_1,\ldots,z_m >) & \hookrightarrow & \mathbf{FF}(\sqrt[P^*]{\mathrm{H}^*} < z_1,\ldots,z_m >). \end{array}$$

Hence, keeping in mind that H^* and $\mathrm{H}^* < z_1, \ldots, z_m >$ are Noetherian, we have

$$\begin{aligned} \operatorname{trzdeg}(\mathbb{F}(z_1,\ldots,z_m)/\mathbb{F}) &= m \\ &\overset{(1)}{=} \dim(\mathrm{H}^*) \\ &\overset{(2)}{=} \dim(\mathrm{H}^* < z_1,\ldots,z_m >) \\ &= \operatorname{trzdeg}(\mathbf{FF}(\mathrm{H}^* < z_1,\ldots,z_m >)/\mathbb{F}), \end{aligned}$$

where (1) is true by assumption and (2) by Lemma 3. Hence the first field extension in the preceding diagram is algebraic.

The second field extension is also algebraic, because $\mathrm{H}^* \hookrightarrow \sqrt[P^*]{\mathrm{H}^*}$ is integral. Therefore Lemma 3.1.1 yields an integral ring extension ...

The rest of Case 1 of the proof is fine. But in Case 2 the wrong Lemma 5.3.1 is again applied. So we have to do some more work: We start in line -13 of Page 96.

... Recall again the constructions from Chapter 5: If $z_1, \ldots, z_m$ is a basis of the vector space over $\mathbb{F}$ of the roots of the Δ-polynomial of $\sqrt[P^*]{\mathrm{H}^*}$ then by Lemma 2

$$Un\,(\mathbf{FF}(\sqrt[P^*]{\mathrm{H}^*} < z_1,\ldots,z_m >))$$

is P^*-inseparably closed. By Lemma 3 all algebras of the chain

$$\mathrm{H}^* \hookrightarrow \mathrm{H}^* < z_1,\ldots,z_m > \hookrightarrow \sqrt[P^*]{\mathrm{H}^*} < z_1,\ldots,z_m >$$

have the same Krull dimension n. However, Theorem 5.3.3 shows that

$$\sqrt[P^*]{\mathrm{H}^*} < z_1,\ldots,z_m > \hookrightarrow Un\,(\mathbf{FF}(\sqrt[P^*]{\mathrm{H}^*} < z_1,\ldots,z_m >)) = \mathbb{F}[z_1,\ldots,z_m],$$

i.e., the unstable part $Un\,(\mathbf{FF}(\sqrt[P^*]{\mathrm{H}^*} < z_1, \ldots, z_m >))$ has Krull dimension $m < n$. This is a contradiction, because by Lemma 1

$$\mathbf{FF}(\sqrt[P^*]{\mathrm{H}^*} < z_1,\ldots,z_m >)) = \mathbf{FF}(Un\,(\mathbf{FF}(\sqrt[P^*]{\mathrm{H}^*} < z_1,\ldots,z_m >)))$$

have the same transcendence degree. •

PAGE 96 : Corollary 6.1.4 is wrong. The ‘‘only if’’ part is ok by Theorem 5.3.5′. However Example 1 of Section 7.4 is a counter example to the ‘‘if’’ part. Indeed, in the very last line of the proof Theorem 5.3.5 is used to conclude from the P^*-inseparable closedness of A^* the P^*-inseparable closedness of H^* which is wrong.

PAGE 118 : Example 1 has some typos: The correct definition of H^* is

$$H^* := \mathbb{F}[x^2 + y^2,\ xy,\ xy(x+y)] / \left((x^2+y^2)(xy)^2 + (xy(x+y))^2\right).$$

The field of fractions of the Dickson algebra is

$$\mathbb{F}(x^2y + xy^2, x^2 + y^2 + xy).$$

Furthermore, it is worthwhile noting that the Δ-length of H^* is two.

PAGE 128 : The last sentence of the statement of Theorem 8.1.5 should read as follows:

If H^* is P^*-inseparably closed then $r = 0$.

PAGE 138 : Example 1 is just fine. Note that the element $x \in H^*$ is a Thomclass but H^* is not reduced. So, this is a counterexample to the reverse Landweber-Stong conjecture, but not to Proposition 8.4.1.

PAGE 145ff : In Appendix A.2 we should replace the definition of the Dickson classes by the following (cf. line -6 page 145):

$$\mathbf{d}_{n,i} = (-1)^{\dim(V^*) - \dim(W^*)} \sum_{W^* \le V^*,\ \dim(W^*) = i} \left(\prod_{v \notin W^*} v \right).$$

This causes some sign changes in the following; in line -3 of page 145:

$$f(X) = \prod_{v \in V^*} (X + v) = X^{q^n} + (-1)^{n-i} \sum_{i=0}^{n-1} \mathbf{d}_{n,i} X^{q^i}.$$

PAGE 147/148 : The sign change above causes the following changes of the formulae of Proposition A.2.1 (and the corresponding formulae on the preceding page 147:

$$\mathcal{P}^k(\mathbf{d}_{n,i}) = \begin{cases} 0 & \text{if } k \ge q^n \\ -\mathcal{P}^{k-q^{n-1}}(\mathbf{d}_{n,n-1})\mathbf{d}_{n,i} + \mathcal{P}^{k-q^{i-1}}(\mathbf{d}_{n,i-1}) & \text{if } 1 \le k < q^n \\ \mathbf{d}_{n,i} & \text{if } k = 0. \end{cases}$$

and

$$\mathcal{P}^{\Delta_k}(\mathbf{d}_{n,i}) = \begin{cases} 0 & \text{if } 0 \le k < n \text{ and } k \ne i \\ (-1)^{i+1}\mathbf{d}_{n,0} & \text{if } 0 \le k < n \text{ and } k = i \\ (-1)^n \mathbf{d}_{n,0}\mathbf{d}_{n,i} & \text{if } k = n \\ \mathcal{P}^{q^{k-1}}\mathcal{P}^{\Delta_{k-1}}\left(\mathbf{d}_{n,i}\right) & \text{if } k > n. \end{cases}$$

In Corollary A.2.2 we have then:

$$\mathcal{P}^{q^k}(\mathbf{d}_{n,i}) = \begin{cases} -\mathbf{d}_{n,i-1} & \text{for } k = i-1 \ge 0 \\ -\mathbf{d}_{n,i}\mathbf{d}_{n,n-1} & \text{for } k = n-1 \ge 0 \\ 0 & \text{otherwise} \end{cases}$$

for all $i = 0, \ldots, n-1$.

References

[1] Mara D. Neusel, *Localizations over the Steenrod Algebra*, Mathematische Zeitschrift 235 (2000), 353-378.