Math 4362 - Number Theory
Homework 4
Due in Class - Thursday October 11, 2018

1. Using Fermat's little theorem, find the remainder on dividing

(a) $1865^{1910} + 1986^{2061}$; and

(b) $2222^{5555} + 5555^{2222}$,

by 7.

2. Use Wilson's Theorem to find the remainder when

(a) 15! is divided by 17.

(b) $2(26!)$ is divided by 29.

3. Find two solutions to the quadratic congruence $x^2 \equiv -1 \pmod{29}$.

4. Calculate $\tau(5040), \sigma(5040)$, and $\mu(5040)$.

5. Prove that $\tau(n)$ is odd if and only if n is a perfect square.

6. Prove that for any positive integer n,

$$\sum_{d|n} 1/d = \sigma(n)/n.$$

7. Let $n = p_1^{a_1} p_2^{a_2} \cdots p_r^{a_r}$ be the prime factorization of a positive integer $n > 1$. If f is a non-zero multiplicative function, prove that

$$\sum_{d|n} \mu(d) f(d) = (1 - f(p_1))(1 - f(p_2)) \cdots (1 - f(p_r)).$$

8. Using Q7, prove that if $n = p_1^{a_1} p_2^{a_2} \cdots p_r^{a_r} > 1$ then

$$\sum_{d|n} \mu(d)\sigma(d) = (-1)^r p_1 p_2 \cdots p_r.$$