Math 4362 - Number Theory
Homework 3
Due in Class - Thursday 20 September 2018

1. If p is a prime and $p \mid a^n$, prove that $p^n \mid a^n$.

2. Find the prime factorization of each of the following numbers:

(a) 288

(b) 14520

(c) 21357

3. Using your results from Q2:

(a) Write down all the divisors of 288; and

(b) Calculate $\gcd(288, 14520)$ and $\text{lcm}(288, 14520)$.

4. **(a)** Using the Division Algorithm, show that all primes $p \geq 5$ have the form $6k+1$ or $6k+5$.

(b) Using part (a), show that for $p \geq 5$ prime, p^2+2 is composite.

5. Let $n = p_1^{a_1} p_2^{a_2} \cdots p_r^{a_r}$ be the prime factorization of some positive integer $n > 1$. Prove that n is a square if and only if $a_1, a_2, \cdots a_r$ are all even.

6. If $p \neq 5$ is an odd prime, prove that either $p^2 - 1$ or $p^2 + 1$ is divisible by 10.