

Review for Exam III

1. For the following matrices, find a basis for the row space, a basis for the column space and a basis for the nullspace. Determine the rank and nullity of each matrix.

$$\text{a) } \begin{pmatrix} 1 & 1 & 1 & 1 & 0 \\ 2 & 3 & -1 & -1 & 2 \\ 3 & 2 & 1 & 1 & 3 \\ 3 & 6 & -1 & -1 & 1 \end{pmatrix} \quad \text{b) } \begin{pmatrix} 1 & 2 & -3 & 1 & 1 \\ -1 & -1 & 4 & -1 & 6 \\ -2 & -4 & 7 & -1 & 1 \end{pmatrix} \quad \text{c) } \begin{pmatrix} 1 & -3 & 1 & 1 \\ 2 & 1 & -1 & 2 \\ 1 & 4 & -2 & 1 \\ 5 & -8 & 2 & 5 \end{pmatrix} \quad \text{d) } \begin{pmatrix} 1 & 1 & -1 \\ 4 & -3 & 3 \end{pmatrix}$$

2. Find a basis for the span of the following set of vectors:

$$\begin{aligned} \text{a) } & \{(1, -1, 2, 5)^T, (0, -1, 3, 1)^T, (3, -4, 9, 16)^T, (1, 1, 0, 0)^T\} \\ \text{b) } & \{(1, 1)^T, (-1, 1)^T, (3, 2)^T\} \\ \text{c) } & \{(1, -1, 1)^T, (2, 0, 1)^T, (4, -2, 3)^T\} \end{aligned}$$

3. Decide whether the following vectors span the given vector space V :

$$\begin{aligned} \text{a) } & \{(1, 2)^T, (2, 4)^T, (5, 10)^T\}, V = \mathbb{R}^2 \\ \text{b) } & \{(1, 1)^T, (0, 1)^T, (-2, 2)^T\}, V = \mathbb{R}^2 \\ \text{c) } & \{x, x + x^2, -x^2\}, V = P_3 \\ \text{d) } & \left\{ \begin{pmatrix} 1 & 1 \\ 0 & 1 \end{pmatrix}, \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}, \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix}, \begin{pmatrix} 0 & 0 \\ 1 & 0 \end{pmatrix} \right\}, V = \mathbb{R}^2 \end{aligned}$$

4. Decide whether the following vectors are linearly dependent in the given vector space. If they are dependent find a nontrivial linear combination that gives the zero vector. If they are independent justify your answer.

$$\begin{aligned} \text{a) } & \{(1, 2, 4)^T, (2, 1, 3)^T, (4, -1, 1)^T\} \text{ in } \mathbb{R}^3 \\ \text{b) } & \{x^2 - 2x + 3, 2x^2 + x + 8, x^2 + 8x + 7\} \text{ in } P_3 \\ \text{c) } & \left\{ \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}, \begin{pmatrix} 0 & 1 \\ 0 & 0 \end{pmatrix}, \begin{pmatrix} 2 & 3 \\ 0 & 2 \end{pmatrix} \right\} \text{ in } \mathbb{R}^4 \end{aligned}$$

5. Determine whether the following vectors form a basis for the given vector space. Justify your answer.

$$\begin{aligned} \text{a) } & \{1 - x, 1 + x, 1 - x^2\} \text{ in } P_3 \\ \text{b) } & \{(-1, 1, 0, 0)^T, (0, -1, 1, 0)^T, (0, 0, -1, 1)^T, (-1, 0, 0, 1)^T\} \text{ in } \mathbb{R}^4 \\ \text{c) } & \{(2, 1, 0)^T, (1, 2, 3)^T, (0, 0, -1)^T\} \text{ in } \mathbb{R}^3 \end{aligned}$$

6. Given the ordered bases

$$\begin{aligned} U &= [\mathbf{u}_1, \mathbf{u}_2, \mathbf{u}_3] = [(1, 1, 1)^T, (1, 2, 2)^T, (2, 3, 4)^T] \\ V &= [\mathbf{v}_1, \mathbf{v}_2, \mathbf{v}_3] = [(4, 6, 7)^T, (0, 1, 1)^T, (0, 1, 2)^T] \end{aligned}$$

find a matrix P such that $[x]_U = P[x]_V$. If $x = \mathbf{v}_1 - 2\mathbf{v}_2 + \mathbf{v}_3$, find the coordinates of x in the ordered basis U .

7. Problems 3-7 from homework 9.

8. Decide whether the given vector \mathbf{b} is in the column space of the matrix A .

a) $A = \begin{pmatrix} 1 & 2 \\ 2 & 4 \\ 1 & 2 \end{pmatrix}, \mathbf{b} = \begin{pmatrix} 5 \\ 10 \\ 5 \end{pmatrix}$

b) $A = \begin{pmatrix} 2 & 3 & 1 \\ 1 & 1 & 1 \\ 3 & 4 & 2 \end{pmatrix}, \mathbf{b} = \begin{pmatrix} 1 \\ 3 \\ 4 \end{pmatrix}$

c) $A = \begin{pmatrix} -1 & 2 & -1 \\ -2 & 2 & 1 \\ 3 & 2 & 2 \\ -3 & 8 & 5 \end{pmatrix}, \mathbf{b} = \begin{pmatrix} 3 \\ 8 \\ -1 \\ 0 \end{pmatrix}$

9. Decide whether each of the following vector space mappings is a linear transformation. If it is linear prove it and find its standard matrix. If not linear show what fails.

a) $L : \mathbb{R} \rightarrow \mathbb{R}$, given by $L(x) = x + 1$

b) $L : \mathbb{R}^3 \rightarrow \mathbb{R}^4$, given by $L((x_1, x_2, x_3)^T) = (x_1, -x_3, x_1 + x_2 + x_3, 0)^T$

c) $L : \mathbb{R}^2 \rightarrow \mathbb{R}^2$, given by $L((x, y)^T) = (xy, x - y)^T$