Homework # 1 Math 2450

Due: September 1, 2015

1. Let $\vec{v} = 4\vec{i} - 2\vec{j} + \vec{k}$ and $\vec{w} = 2\vec{j} + 3\vec{k}$. Compute the following:
   (a) $\vec{v} \cdot \vec{w}$
   (b) $\vec{v} \times \vec{w}$
   (c) A unit vector orthogonal to both $\vec{v}$ and $\vec{w}$.
   (d) The direction cosines of $\vec{v}$.
   (e) The angle between $\vec{v}$ and $\vec{w}$.
2. Write an equation for a sphere with center $(1, 2, 3)$ and passing through the point $(2, -1, 5)$.
3. Graph the cylinder and the plane: $x^2 + z^2 = 25$ and $x + 2y = 4$.
4. Suppose that a wind is blowing with a 1000-lb magnitude force $\vec{F}$ in the direction $N60^oW$ behind a boat's sail. How much work does the wind perform in moving the boat in a northerly direction a distance of 50 feet? Express your answer in foot-pounds.
5. Find the volume of the parallelpiped formed by the three vectors $\vec{u} = \vec{i} - \vec{j}$, $\vec{v} = 2\vec{i} - \vec{k}$ and $\vec{w} = 3\vec{j} + \vec{k}$.
6. Let $\vec{v} = \langle 1, 1, 1\rangle$. Find all vectors $\vec{w}$ such that $\vec{v} \times \vec{w} = \vec{w}$.
7. Let $\vec{u} = \vec{i} + \vec{j}$, $\vec{v} = 2\vec{i} - \vec{j} + \vec{k}$ and $\vec{w} = 4\vec{i}$. Compute $(\vec{u} \times \vec{v}) \times \vec{w}$ and $\vec{u} \times (\vec{v} \times \vec{w})$ to show that the cross product is NOT associative.
8. For the following lines in $\mathbb{R}^3$, compute
   (a) parametric form passing through $(3, 2, -2)$ and parallel to both the $xy-$ and $yz-$ planes.
   (b) symmetric form passing through $(-2, 2, 5)$ and $(2, 0, -4)$.
   (c) Find two unit vectors parallel to the line: $\dfrac{x-2}{4} = \dfrac{y}{2} = z + 1$.
   (d) parametric equations for a line passing through $(1, 2, 3)$ and perpendicular to the plane $-2x - y + 2z = 1$.
9. Sketch the path described by the parametric equations.
   (a) $x = t + 1$, $y = t^2 - 2$, $-1 \leq t \leq 2$
   (b) $x = 2 + 3\cos\theta$, $y = -4 + 5\sin(\theta)$, $0 \leq \theta \leq 2\pi$
   (c) $x = \exp(t)$, $y = \exp(-t)$, $-\infty < t < \infty$.
10. Write an equation for a plane in standard form $Ax + By + Cz + D = 0$:
   (a) passing through the point $(2, 5, 0)$ with normal vector $\vec{N} = 2\vec{i} + 4\vec{k}$.
   (b) passing through the points $(2, 1, 1)$, $(1, 3, 0)$ and $(-4, 0, 2)$.