Answers to Review Problems

1 (a) $\cos(\theta) = \dfrac{\vec{u}\cdot\vec{v}}{\|\vec{u}\|\|\vec{v}\|} = \dfrac{-2+1+6}{\sqrt{14}\sqrt{6}} = \dfrac{5}{2\sqrt{21}}$ $\quad \boxed{\theta = \cos^{-1}\left(\frac{5}{2\sqrt{21}}\right)}$

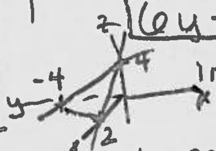
(b) $\langle 1,0,0\rangle$ or $\langle 0, \frac{-3}{\sqrt{10}}, \frac{1}{\sqrt{10}}\rangle$ or $\langle \frac{1}{\sqrt{11}}, \frac{3}{\sqrt{11}}, \frac{-1}{\sqrt{11}}\rangle$ etc.

(c) $\left|\begin{vmatrix} -2 & 1 & 3 \\ 1 & 1 & 2 \\ 0 & 1 & 3 \end{vmatrix}\right| = |-2(1) - 1(3) + 3(1)| = |-2|$ $\quad$ Volume $= \boxed{2}$

(d) $\vec{u} \times \vec{w} = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ -2 & 1 & 3 \\ 0 & 1 & 3 \end{vmatrix} = \vec{i}(0) - \vec{j}(-6) + \vec{k}(-2) = 6\vec{j} - 2\vec{k} = \vec{N}$

$6y - 2z + D = 0$
$6(3) - 2(-2) + D = 0$
$18 + 4 + D = 0$
$D = -22$

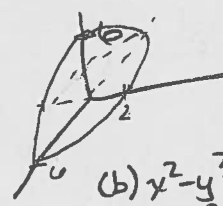
$\boxed{6y - 2z - 22 = 0}$



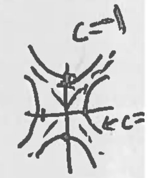
Intercepts of the plane

2 (a) $z = y - 2x + 4$, plane

$z = \sqrt{36 - x^2 - 9y^2}$, half an ellipsoid $\quad \dfrac{z^2}{36} + \dfrac{y^2}{4} + \dfrac{x^2}{36} = 1$



(b) $x^2 - y^2 = 1$
$y^2 - x^2 = 1$



3. $d = \dfrac{|Ax_1 + By_1 + Cz_1 + D|}{\sqrt{A^2 + B^2 + C^2}} = \dfrac{|2+3+8-1|}{\sqrt{1^2 + 1^2 + 2^2}} = \boxed{\dfrac{12}{\sqrt{6}}}$

4. (a) $\vec{v}_1 = \langle 2, -3, 0\rangle$
$\vec{v}_2 = \langle -1, -1, 1\rangle$ Not parallel

$x = 2t \quad y = 1 - 3t \quad z = 2$

$x = 1 - s \quad y = 2 - s \quad z = 1 + s$

solve for s, t to see if there is a unique sol if NO $\Rightarrow$ skew.

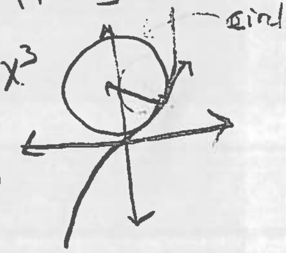
$\left.\begin{array}{l} 2t = 1 - s \\ 1 - 3t = 2 - s \\ 2 = 1 + s \end{array}\right\} \Rightarrow \begin{array}{l} t = 0 \\ s = 1 \end{array} \Rightarrow \left.\begin{array}{l} x_0 = 0 \\ y_0 = 1 \\ z_0 = 2 \end{array}\right\} t=0 \quad \left.\begin{array}{l} x_0 = 0 \\ y_0 = 1 \\ z_0 = 2 \end{array}\right\} s=1$ $\quad$ $\boxed{\text{Intersect at } (0,1,2)}$

(b) $\langle 1, 2, -1\rangle = \vec{N}_1$
$\langle 2, 0, 2\rangle = \vec{N}_2$
$\vec{N}_1 \cdot \vec{N}_2 = 0 \Rightarrow$ planes are perpendicular

(c) $\boxed{\begin{array}{l} x = 3 + 2t \\ y = 3 - 5t \\ z = -2 - t \end{array} \quad -\infty < t < \infty}$

5. $\|\vec{V}(t)\| = \sqrt{\frac{9}{t^2} + 16e^{8t}}$ $\quad \vec{A}(t) = \frac{-3}{t^2}\vec{i} + 16e^{4t}\vec{k}$, $\quad \vec{R}(t) = 3\ln|t|\vec{i} + \vec{j} + (e^{4t} - e^4)\vec{k}$

6. $\vec{T}(t) = \dfrac{1}{\sqrt{1+9t^4}}\vec{i} + \dfrac{3t^2}{\sqrt{1+9t^4}}\vec{j}$ $\quad \vec{N} = \dfrac{-3t^2}{\sqrt{1+9t^4}}\vec{i} + \dfrac{1}{\sqrt{1+9t^4}}\vec{j}$



$\kappa = \dfrac{\|\vec{R}'(t) \times \vec{R}''(t)\|}{\|\vec{R}'(t)\|^3} = \dfrac{|6t|}{\left(\sqrt{1+9t^4}\right)^3} \Rightarrow$ at $t=1$ $\boxed{\kappa = \dfrac{6}{10\sqrt{10}}}$ $\quad r = \dfrac{10\sqrt{10}}{6}$

Circle center $\langle x_0, y_0\rangle = \langle 1,1\rangle + r\vec{N}(1) = \langle 1,1\rangle + \frac{10\sqrt{10}}{6}\langle \frac{-3}{\sqrt{10}}, \frac{1}{\sqrt{10}}\rangle = \langle 1-5, 1+\frac{5}{3}\rangle$
$= \langle -4, \frac{8}{3}\rangle$

$$\boxed{(x+4)^2 + \left(y - \frac{8}{3}\right)^2 = \frac{250}{9}}$$

7. $S(t)=\int_{t_0}^{t}\|\vec{R}'(t)\|dt \qquad \vec{R}'(t)=\langle e^t,\ e^t(\cos(\pi t)-\pi\sin(\pi t)),\ e^t(\sin(\pi t)+\pi\cos(\pi t))\rangle$

$$\|\vec{R}'(t)\| = \sqrt{e^{2t}+e^{2t}(\cos^2(\pi t)-2\pi\cos(\pi t)\sin(\pi t)+\pi^2\sin^2(\pi t)) + e^{2t}(\sin^2(\pi t)+2\pi\sin(\pi t)\cos(\pi t)+\pi^2\cos^2(\pi t))}$$

$$= \sqrt{e^{2t}\left(1+\cos^2(\pi t)+\sin^2(\pi t)+\pi^2[\sin^2(\pi t)+\cos^2(\pi t)]\right)}$$

$$= \sqrt{e^{2t}(2+\pi^2)} = e^t\sqrt{2+\pi^2}$$

Length $t=0$ to $t=2$ $\quad S=\int_0^2 e^t\sqrt{2+\pi^2}\,dt = e^t\sqrt{2+\pi^2}\Big|_0^2 = \boxed{(e^2-1)\sqrt{2+\pi^2}}$

8. (a) $\lim\limits_{(x,y)\to(2,1)}(x^2+4y^2) = 4+4=8$

(b) $y=x$: $\lim\limits_{x\to 0}\frac{3x^2\cdot x^2}{x^4+x^4} = \lim\limits_{x\to 0}\frac{3x^4}{2x^4}=\frac{3}{2}$

$y=2x$: $\lim\limits_{x\to 0}\frac{3x^2(2x)^2}{x^4+(2x)^4} = \lim\limits_{x\to 0}\frac{12x^4}{x^4+16x^4} = \frac{12}{17}$

The limit is <u>not</u> unique.

$\frac{3}{2}\neq\frac{12}{17}$

9. $f_x = ye^{xy}\cos(y) \qquad f_{xx}=y^2e^{xy}\cos(y)$

$f_y = e^{xy}(x\cos(y)-\sin(y)) \qquad f_{xy}=e^{xy}(\cos(y)+xy\cos(y)-y\sin(y))$

$f_{yy}=e^{xy}(x^2\cos(y)-2x\sin(y)-\cos(y))$