Math 2450.H01, Quiz 3
Fall 2015

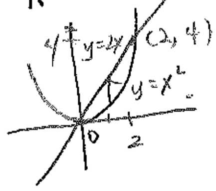
Show all of your work. Circle your answers.
No calculators. Turn off all electronic devices.

Name Answers

1. (16 pts) Evaluate the integrals.

(a) $\int_1^2 \int_0^1 (x+y)\, dx\, dy$ $= \int_1^2 \left(\frac{x^2}{2} + yx\right)\Big|_{x=0}^{x=1} dy = \int_1^2 \left(\frac{1}{2}+y\right) dy = \frac{1}{2}y + \frac{y^2}{2}\Big|_1^2 = (1+2) - \left(\frac{1}{2}+\frac{1}{2}\right)$
$= \boxed{2}$

(b) $\iint_R x^2 y\, dy\, dx$, where R is the region in the x-y plane bounded by $y = x^2$ and $y = 2x$. Sketch R.



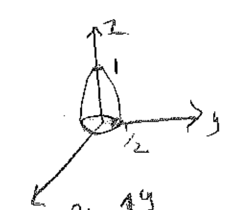
$\int_0^2 \int_{x^2}^{2x} x^2 y\, dy\, dx = \int_0^2 x^2 \frac{y^2}{2}\Big|_{y=x^2}^{y=2x} dx = \int_0^2 x^2 \cdot \frac{4x^2}{2} - \frac{x^2 x^4}{2}\, dx$

$= \int_0^2 \left(2x^4 - \frac{x^6}{2}\right) dx = \frac{2}{5}x^5 - \frac{x^7}{14}\Big|_0^2 = \frac{2^6}{5} - \frac{2^7}{14}$

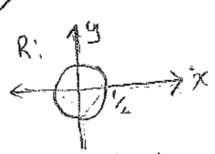
$= \frac{2^6}{5} - \frac{2^6}{7} = \frac{2^6}{35}(7-5) = \frac{2^7}{35} = \boxed{\frac{128}{35}}$

$x^2 = 2x$
$x^2 - 2x = 0$
$x(x-2) = 0$
$x = 0, 2$

2. (8 pts) Find the volume D which is bounded by the paraboloid $z = 1 - 4(x^2+y^2)$ and the x-y plane.



R:



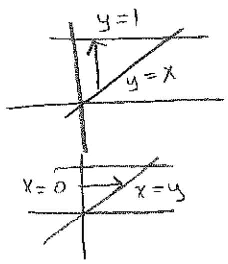
$x^2+y^2 = \frac{1}{4}$

$\iint_R \int_0^{1-4(x^2+y^2)} dz\, dA = \int_0^{2\pi}\int_0^{\frac{1}{2}}\int_0^{1-4r^2} dz\, r\, dr\, d\theta = \int_0^{2\pi}\int_0^{\frac{1}{2}} \underbrace{(1-4r^2)r}_{r-4r^3}\, dr\, d\theta$

Change to polar coordinates

$= \int_0^{2\pi} \left(\frac{r^2}{2} - \frac{4r^4}{4}\right)\Big|_{r=0}^{r=\frac{1}{2}} d\theta = \int_0^{2\pi}\left(\frac{1}{8} - \frac{1}{16}\right) d\theta = \int_0^{2\pi} \frac{1}{16}\, d\theta = \frac{2\pi}{16} = \boxed{\frac{\pi}{8}}$

3. (9 pts) Change the order of integration, then evaluate the integral $\int_0^1 \int_x^1 \cos(y^2)\,dy\,dx = \int_A^B \int_C^D \cos(y^2)\,dx\,dy$



$$\int_0^1\int_0^y \cos(y^2)\,dx\,dy = \int_0^1 x\cos(y^2)\Big|_{x=0}^{x=y} dy$$

$$= \int_0^1 y\cos(y^2)\,dy = +\frac{1}{2}\sin(y^2)\Big|_{y=0}^{y=1} = \boxed{\frac{1}{2}\sin(1)}$$

substitution
$u = y^2$
$du = 2y\,dy$

4. (9 pts) Compute the surface area of the plane $x - 5y + z = 10$ that lies inside the cylinder $x^2 + y^2 = 4$, where $x \geq 0$ and $y \geq 0$

$z = -x + 5y + 10 = f(x,y)$
$f_x = -1 \quad f_y = 5$

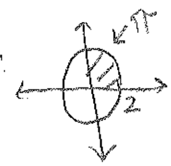
$$\iint_R \sqrt{(-1)^2 + (+5)^2 + 1}\,dA$$

$= \sqrt{27} \cdot$ Area of R

$= \sqrt{27}\,\pi$

$= \boxed{3\sqrt{3}\pi}$

R:



or $\int_0^{\pi/2}\int_0^2 \sqrt{27}\,r\,dr\,d\theta$

$= \int_0^{\pi/2} \sqrt{27}\frac{r^2}{2}\Big|_0^2 d\theta = 2\sqrt{27}\cdot\frac{\pi}{2} = \sqrt{27}\pi$

$= \boxed{3\sqrt{3}\pi}$

5. (8 pts) Evaluate $\iiint_D \underbrace{(x^2 + y^2 + z^2)^{3/2}}_{(\rho^2)^{3/2} = \rho^3}\,dx\,dy\,dz$, where D is defined by $x^2 + y^2 + z^2 \leq 6$.

radius of sphere is $\rho = \sqrt{6}$

$$\int_0^{2\pi}\int_0^{\pi}\int_0^{\sqrt{6}} \underbrace{\rho^3 \rho^2 \sin\phi}_{\rho^5 \sin\phi}\,d\rho\,d\phi\,d\theta = \int_0^{2\pi}\int_0^{\pi} \frac{\rho^6}{6}\Big|_0^{\sqrt{6}} \sin\phi\,d\phi\,d\theta$$

$$= 36\int_0^{2\pi} -\cos\phi\Big|_0^{\pi} d\theta$$

$$= 36\int_0^{2\pi} \underbrace{(-\cos(\pi) + \cos(0))}_{2}\,d\theta$$

$72 \cdot 2\pi = \boxed{144\pi}$