Solve Laplace's equation on a Rectangular Domain
Due: Wednesday, October 24, 2018

1. (16 pts) Solve Laplace's equation $u_{xx}+u_{yy}=0$ on the square $0<x<1$, $0<y<1$ with the following boundary conditions.

   (a) $u(0,y)=0$, $u(1,y)=0$, $0<y<1$
   $u(x,0)=2\sin(3\pi x)$, $u(x,1)=0$, $0<x<1$.
   What is the maximum temperature on the square?
   What is the minimum temperature on the square?

   (b) $u(0,y)=0$, $u(1,y)=\sin(\pi y)$, $0<y<1$
   $u(x,0)=0$, $u(x,1)=0$, $0<x<1$.

   (c) $u(0,y)=0$, $u(1,y)=\sin(\pi y)$, $0<y<1$
   $u(x,0)=2\sin(3\pi x)$, $u(x,1)=0$, $0<x<1$.

2. (4 pts) The solution of Laplace's equation $u_{xx} + u_{yy} = 0$ on the square $0 < x < \pi$, $0 < y < \pi$ is

$$u(x,y) = \frac{\sinh(2y)}{\sinh(2\pi)}\sin(2x) - 2\frac{\sinh(4y)}{\sinh(4\pi)}\sin(4x) + \left[3\cosh(x) - \frac{3\cosh(\pi)}{\sinh(\pi)}\sinh(x)\right]\sin(y).$$

What is the temperature on each of the four boundaries, $x = 0$? $x = \pi$? $y = 0$? $y = \pi$? The solution is graphed below.