ANSWERS

In-Class Assignment # 2
Separable Partial Differential Equations
Wednesday, October 3, 2018

1. (10 pts) Classify the following second-order partial differential equations as hyperbolic, parabolic or elliptic. $u \equiv u(x, y)$.

(a) $\dfrac{\partial^2 u}{\partial x^2} + \dfrac{\partial^2 u}{\partial y^2} = 0.$ Discriminant: $B^2-4AC = 0-4(1)(1) < 0 \Rightarrow$ elliptic
$A=1, B=0, C=1$

(b) $\dfrac{\partial^2 u}{\partial x^2} + 2\dfrac{\partial^2 u}{\partial x \partial y} + \dfrac{\partial^2 u}{\partial y^2} = 0$ Discriminant: $B^2-4AC = 4-4(1)(1) = 0 \Rightarrow$ parabolic
$A=1, B=2, C=1$

(c) $\dfrac{\partial^2 u}{\partial x \partial y} = u$ Discriminant: $B^2-4AC = 1^2-4(0)(0) = 1 > 0 \Rightarrow$ hyperbolic
$A=0, B=1, C=0$

2. (10 pts) For problems (a), (b), and (c), determine whether the partial differential equations are separable. If they are separable, separate the variables.

(a) $\dfrac{X''Y}{XY} + \dfrac{XY''}{XY} = \dfrac{0}{XY} \Rightarrow \dfrac{X''}{X} + \dfrac{Y''}{Y} = 0 \Rightarrow \dfrac{X''}{X} = -\dfrac{Y''}{Y}$ separable

(b) $\dfrac{X''Y}{Y} + 2\dfrac{X'Y'}{Y} + \dfrac{XY''}{Y} = 0 \Rightarrow X'' + 2\dfrac{X'Y'}{Y} + \dfrac{XY''}{Y} = 0$

$\Rightarrow \dfrac{X''}{X} + \dfrac{2X'Y'}{XY} + \dfrac{XY''}{XY} \Rightarrow \dfrac{X''}{X} + \dfrac{2X'Y'}{XY} + \dfrac{Y''}{Y} = 0$ not separable

(c) $\dfrac{X'Y'}{XY} = \dfrac{XY}{XY} \Rightarrow \dfrac{X'Y'}{XY} = 1 \Rightarrow \dfrac{X'}{X} = \dfrac{Y}{Y'} \Rightarrow$ separable