Fourier Series, Fourier Cosine Series and Fourier Sine Series

Formulas for the expansion of a function $f(x)$ on $[-L, L]$ as a Fourier series and Fourier sine series or Fourier cosine series on $[0, L]$.

Fourier series of $f(x)$ on $[-L, L]$:

$$\begin{aligned} f(x) &= \frac{a_0}{2} + \sum_{n=1}^{\infty} \left[a_n \cos\left(\frac{n\pi x}{L}\right) + b_n \sin\left(\frac{n\pi x}{L}\right) \right] \\ a_0 &= \frac{1}{L} \int_{-L}^{L} f(x)\, dx \\ a_n &= \frac{1}{L} \int_{-L}^{L} f(x) \cos\left(\frac{n\pi x}{L}\right) dx, \ n = 1, 2, \ldots, \\ b_n &= \frac{1}{L} \int_{-L}^{L} f(x) \sin\left(\frac{n\pi x}{L}\right) dx, \ n = 1, 2, \ldots. \end{aligned}$$

Fourier cosine series of $f(x)$ on $[0, L]$ for an EVEN function:

$$\begin{aligned} f(x) &= \frac{a_0}{2} + \sum_{n=1}^{\infty} a_n \cos\left(\frac{n\pi x}{L}\right) \\ a_0 &= \frac{2}{L} \int_{0}^{L} f(x)\, dx \\ a_n &= \frac{2}{L} \int_{0}^{L} f(x) \cos\left(\frac{n\pi x}{L}\right) dx, \ n = 1, 2, \ldots. \end{aligned}$$

Fourier sine series of $f(x)$ on $[0, L]$ for an ODD function:

$$\begin{aligned} f(x) &= \sum_{n=1}^{\infty} b_n \sin\left(\frac{n\pi x}{L}\right) \\ b_n &= \frac{2}{L} \int_{0}^{L} f(x) \sin\left(\frac{n\pi x}{L}\right) dx, \ n = 1, 2, \ldots. \end{aligned}$$