Classify (0,0) as node, center, spiral, or saddle and stable or unstable.

11.2

4. $\mathbf{A} = \begin{pmatrix} -1 & -4 \\ 1 & -1 \end{pmatrix}$,

$$\mathbf{X}(t) = e^{-t}\left[c_1\begin{pmatrix} 2\cos 2t \\ \sin 2t \end{pmatrix} + c_2\begin{pmatrix} -2\sin 2t \\ \cos 2t \end{pmatrix}\right]$$

5. $\mathbf{A} = \begin{pmatrix} -6 & 5 \\ -5 & 4 \end{pmatrix}$,

$$\mathbf{X}(t) = c_1\begin{pmatrix} 1 \\ 1 \end{pmatrix}e^{-t} + c_2\left[\begin{pmatrix} 1 \\ 1 \end{pmatrix}te^{-t} + \begin{pmatrix} 0 \\ \frac{1}{5} \end{pmatrix}e^{-t}\right]$$

6. $\mathbf{A} = \begin{pmatrix} 2 & 4 \\ -1 & 6 \end{pmatrix}$,

$$\mathbf{X}(t) = c_1\begin{pmatrix} 2 \\ 1 \end{pmatrix}e^{4t} + c_2\left[\begin{pmatrix} 2 \\ 1 \end{pmatrix}te^{4t} + \begin{pmatrix} 1 \\ 1 \end{pmatrix}e^{4t}\right]$$

7. $\mathbf{A} = \begin{pmatrix} 2 & -1 \\ 3 & -2 \end{pmatrix}$, $\mathbf{X}(t) = c_1\begin{pmatrix} 1 \\ 1 \end{pmatrix}e^{t} + c_2\begin{pmatrix} 1 \\ 3 \end{pmatrix}e^{-t}$

8. $\mathbf{A} = \begin{pmatrix} -1 & 5 \\ -1 & 1 \end{pmatrix}$,

$$\mathbf{X}(t) = c_1\begin{pmatrix} 5\cos 2t \\ \cos 2t - 2\sin 2t \end{pmatrix} + c_2\begin{pmatrix} 5\sin 2t \\ 2\cos 2t + \sin 2t \end{pmatrix}$$

In Problems 9–16, classify the critical point (0, 0) of the given linear system ~~by computing the trace τ and determinant Δ and using Figure 11.2.12~~.

9. $x' = -5x + 3y$
   $y' = 2x + 7y$

10. $x' = -5x + 3y$
    $y' = 2x - 7y$

11. $x' = -5x + 3y$
    $y' = -2x + 5y$

12. $x' = -5x + 3y$
    $y' = -7x + 4y$

13. $x' = -\frac{3}{2}x + \frac{1}{4}y$
    $y' = -x - \frac{1}{2}y$

14. $x' = \frac{3}{2}x + \frac{1}{4}y$
    $y' = -x + \frac{1}{2}y$

15. $x' = 0.02x - 0.11y$
    $y' = 0.10x - 0.05y$

16. $x' = 0.03x + 0.01y$
    $y' = -0.01x + 0.05y$

17. Determine conditions on the real constant μ so that (0, 0) is a center for the linear system

$$x' = -\mu x + y$$
$$y' = -x + \mu y.$$