Math 4350.004 Spring 2018
Oral Presentation in Class

1. 1.1 # 19 (a), (b)
2. Prove: The function $f : A \to B$ is a bijection iff for every set $E \subseteq A$, $f(A \setminus E) = B \setminus f(E)$.
3. Identify which of the following statements are false. Then give an example to show why the statement is false.
   (a) Suppose $f : A \to B$ and $H \subseteq B$. Then $H \subseteq f(f^{-1}(H))$.
   (b) Suppose $f : A \to B$ and $E \subseteq A$. Then $f^{-1}(f(E)) \subseteq E$.
   (c) Suppose $f : A \to B$ and f is a surjection such that $y \in B$ and $f(x) = y$. Then $x = f^{-1}(y)$.
4. Give an example of a collection $\{S_n\}_{n=1}^{\infty}$, where each set S_n is denumerable with the following properties: S_1 contains S_2, S_2 contains S_3 and in general, S_n contains S_{n+1}, that is,
   $$S_1 \supseteq S_2 \supseteq S_3 \supseteq \cdots \supseteq S_n \supseteq S_{n+1} \supseteq \cdots \tag{1}$$
   and their intersection is empty, $\bigcap_{n=1}^{\infty} S_n = \emptyset$. (The order of the sets in (1) is called a *nested sequence*.) (YJN)
5. Prove that $0 < |x-1| < 1$ implies $|x^3 + x - 2| < 8|x-1|$. Is this true if $0 \leq |x-1| < 1$? (MPE)
6. 2.1 # 8 (AY)
7. 2.2 # 5 (CNL)
8. 2.2 # 17 (VLL)
9. 2.3 # 7 (SMS)
10. 2.5 # 8 (JLS)
11. 2.5 # 7 & # 9
12. 3.1 # 8 (FJA)
13. 3.1 # 10 (TMR)
14. Give an example of a divergent sequence $X = (x_n)$ such that $\lim(x_{n+1}/x_n) = 1$. (JNB)
15. Give examples of two divergent sequences X and Y whose sum $X + Y$ converges. (BBH)
16. Give examples of two divergent sequences X and Y whose product XY converges. (HAL)
17. 3.2 # 4 (ABG)
18. 3.2 # 18 (ARP)
19. 3.2 # 21 (EST)
20. 3.3 # 8 (TLP)
21. 3.4 # 11 (YJN)
22. 3.4 # 15
23. 3.5 # 8 (SK)
24. 3.5 # 11

25. True or False? If it is true, give a proof. If it is false, give a counterexample.

    (a) If $\sum x_n$ and $\sum y_n$ are both convergent, then $\sum(x_n + y_n)$ is convergent. (True) (SAL)
    (b) If $\sum x_n$ and $\sum y_n$ are both divergent, then $\sum(x_n + y_n)$ is divergent. (False) (KPM-E)
    (c) If $\sum x_n$ is convergent and $\sum y_n$ is divergent, then $\sum(x_n + y_n)$ is divergent.
    (d) If $\sum x_n$ with $x_n > 0$ is convergent, then $\sum x_n^2$ is convergent. (True) (SKK)
    (e) If $\sum x_n$ with $x_n > 0$ is convergent, then $\sum \sqrt{x_n}$ is convergent. (False) (BBH)
    (f) If $\sum |x_n|$ is convergent, then $\sum x_n$ is convergent. (RMV)

26. Give examples of functions f and g, $f : A \to \mathbb{R}$, $g : A \to \mathbb{R}$, such that f and g do not have limits at a point c, but such that both $f + g$ and fg have limits at c. (SKK)

27. Let $f : A \to \mathbb{R}$ and let c be a cluster point of A. If $\lim_{x \to c} f$ exists and if $|f|$ denotes the absolute value function defined for $x \in A$ by $|f|(x) = |f(x)|$, prove $\lim_{x \to c} |f| = \left| \lim_{x \to c} f \right|$.

28. 5.1 # 12

29. 5.3 # 13

30. 5.3 #4 (VLL)

31. 5.3 #5 (CNL)

32. 5.3 #17 (DRD)

33. 5.4 # 5 (SAW)

34. 5.4 #6 (JTM)