Final Exam, Math 2450, Fall 2013

Do not use a calculator. Show all of your work in the Blue Book. Each problem is worth 10 pts.

1. Given the acceleration vector $\vec{A}(t) = \cos(2t)\vec{\imath} + e^{(t-1)}\vec{\jmath} + \dfrac{1}{t^2}\vec{k}$, $\vec{V}(1) = \vec{\jmath}$, and $\vec{R}(1) = 2\vec{\imath}$ determine the velocity vector $\vec{V}(t)$ and position vector $\vec{R}(t)$.
2. (a) Find the unit tangent vector $\vec{T}(t)$ and principal unit normal vector $\vec{N}(t)$ for the curve traced by the position vector $\vec{R}(t) = t\vec{\imath} + \cos(2t)\vec{\jmath} - \sin(2t)\vec{k}$.

   (b) Find the arclength from $t_0 = 0$ to $t_1 = \pi$, $s = \displaystyle\int_{t_0}^{t_1} \|\vec{R}'(t)\|\, dt$.
3. Calculate the following partial derivatives.

   (a) $\dfrac{\partial f}{\partial u}$ if $f(x, y, z) = \sin(z)e^{x^2 y}$, $x = 3u + 4v$, $y = \dfrac{\ln(u)}{v}$ and $z = u^2$.

   (b) $\dfrac{dg}{dt}$ if $g(x, y) = \tan(x^2 + y^2)$, $x = \sqrt{t}$ and $y = \cos(t^2 + 1)$.
4. Given the function $f(x, y) = x^4 + 3y^2 - 2x^2 - 6y - 4$, find the critical points. Then use the second partials test to classify them as relative maxima, relative minima or saddle points.
5. Use Lagrange multipliers to find the minimum of $f(x, y, z) = 2x^2 + 3y^2 + z^2$ subject to the constraint $2x - 3y + z = 5$.
6. Evaluate the integral with the order of integration reversed: $\displaystyle\int_0^2 \int_y^2 \sqrt{1 + x^2}\, dx\, dy$.
7. Write an equation for the tangent plane to the surface $2x^2 - xz + y^2 - z^3 = 1$ at the point $P_0(1, -1, 1)$.
8. Find the volume between the two paraboloids: $z = x^2 + y^2$ and $z = 8 - x^2 - y^2$.
9. Evaluate the double integral: $\displaystyle\int_{-2}^{2} \int_{-\sqrt{4-x^2}}^{\sqrt{4-x^2}} \sqrt{x^2 + y^2}\, dydx$.
10. Evaluate the triple integral: $\displaystyle\iiint_H \sqrt{x^2 + y^2 + z^2}\, dV$, where H is the hemisphere $x^2 + y^2 + z^2 \le 4$ and $x \ge 0$.
11. Evaluate the line integral $\displaystyle\int_C [xy\, dx + x^3 dy]$, where C is the curve $y = x^2 + 1$ from $(0, 1)$ to $(1, 2)$.
12. Use Green's Theorem in the plane to evaluate the line integral $\displaystyle\oint_C [2xydx + 3xdy]$, where C is the boundary of the unit square passing through (0,0), (1,0), (1,1), (0,1), (0,0).
13. Show that the vector field $\vec{F} = (2xy^2 - 4)\vec{\imath} + (2x^2 y + 3y^2)\vec{\jmath}$ is conservative, then find the scalar potential f ($\nabla f = \vec{F}$) and evaluate the line integral $\int_C \vec{F} \cdot d\vec{R}$, where C is the curve $y = x^2 - 1$ from $(1, 0)$ to $(2, 3)$.
14. Make a change of variable to evaluate the double integral $\displaystyle\iint_D (x - 3y)^4 (x + 2y)^9\, dA$, where D is the region $1 \le x - 3y \le 2$ and $0 \le x + 2y \le 1$.
15. Use the Divergence Theorem to evaluate the flux integral $\displaystyle\iint_S \vec{F} \cdot \vec{N}\, dS$, where S is the surface of the sphere $x^2 + y^2 + z^2 = 9$ and $\vec{F} = 2x\vec{\imath} + (y + z^2)\vec{\jmath} + 3z\vec{k}$ ($\vec{N}$ is outward normal).
16. Use Stoke's Theorem to evaluate the line integral $\displaystyle\oint_C [xy\, dx + y^3 dy + z\, dz] = \iint_S \text{curl}\vec{F} \cdot \vec{N} dS$, where S is surface of the paraboloid $z = 9 - x^2 - y^2$ with $z \ge 8$. Curve C the boundary of this surface in the plane $z = 8$, traversed counterclockwise as viewed from above ($\vec{N}$ is upward normal).
17. Answer True or False. Let $f(x, y) = x^2 + y^2$

    (a) The plane tangent to the surface of f at $(1, 1)$ is $z = 2 + 2x(x - 1) + 2y(y - 1)$.

    (b) The directional derivative in the direction from $P_0(1, 1)$ to $P_1(2, 3)$ is $f_{\vec{u}}(1, 1) = 6\sqrt{5}$.

    (c) The largest value of the directional derivative of f at the point (1,1) is $2\sqrt{2}$.