Review Topics
Chapter 13.1, 13.3-13.7

Vector Field: Each point in the plane or in space has a vector $\vec{F}$ associated with it.

Del operator: $\nabla = \vec{i}\frac{\partial}{\partial x} + \vec{j}\frac{\partial}{\partial y} + \vec{k}\frac{\partial}{\partial z}$.

Divergence of $\vec{F}$ is a <u>scalar</u>: $\text{div}\vec{F} = \nabla \cdot \vec{F} = \frac{\partial M}{\partial x} + \frac{\partial N}{\partial y} + \frac{\partial P}{\partial z}$. If $\text{div}\vec{F} = 0$, then vector field is called *incompressible*.

Curl of $\vec{F}$ is a <u>vector</u>: $\text{curl}\vec{F} = \nabla \times \vec{F} = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ M & N & P \end{vmatrix} = \vec{i}\left(\frac{\partial P}{\partial y} - \frac{\partial N}{\partial z}\right) - \vec{j}\left(\frac{\partial P}{\partial x} - \frac{\partial M}{\partial z}\right) + \vec{k}\left(\frac{\partial N}{\partial x} - \frac{\partial M}{\partial y}\right)$.

The $\text{curl}\vec{F}$ gives the axis of rotation. If $\text{curl}\vec{F} = \vec{0}$, then the vector field is *irrotational.*

Conservative vector field if $\vec{F} = \nabla f$, that is, $M\vec{i} + N\vec{j} + P\vec{k} = f_x\vec{i} + f_y\vec{j} + f_z\vec{k}$, $M = f_x$, $N = f_y$, $P = f_z$. Then f is called a **scalar potential**.

(1) Plane: $\vec{F} = M\vec{i} + N\vec{j}$ is **conservative** if the cross partials test holds, $\frac{\partial N}{\partial x} = \frac{\partial M}{\partial y}$

(2) Space: $\vec{F} = M\vec{i} + N\vec{j} + P\vec{k}$ is **conservative** if the curl satisfies $\text{curl}\vec{F} = \vec{0}$.

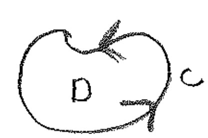
Fundamental Theorem for Line Integrals: If $\vec{F}$ is **conservative**, $\vec{F} = \nabla f$ in a region D containing the curve C with endpoints P to Q, then the line integral is independent of the path C taken from P to Q:

$$\boxed{\int_C \vec{F} \cdot d\vec{R} = \int_C \nabla f \cdot d\vec{R} = f(x,y,z)\Big|_P^Q = f(Q) - f(P).}$$

Green's Theorem in the Plane can be used to evaluate line integrals in the plane over closed curves, $\oint_C (M dx + N dy)$.

If D is a simply connected region in the x-y plane, bounded by a closed curve C traced counterclockwise, then

$$\boxed{\oint_C \vec{F} \cdot d\vec{R} = \oint_C (M dx + N dy) = \iint_D \left(\frac{\partial N}{\partial x} - \frac{\partial M}{\partial y}\right) dA}$$



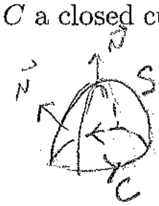
Surface Integral: $\iint_S g(x,y,z)\, dS = \iint_R g(x,y,f(x,y))\sqrt{f_x^2 + f_y^2 + 1}\, dA$, where $z = f(x,y)$ is the surface and R is the projected region in the x-y plane.

Flux Integral: $\iint_S \vec{F} \cdot \vec{N} dS$, flux across the surface oriented by the unit normal field $\vec{N}$.

Stokes' Theorem in Space can be used to evaluate line integrals in space over closed curves, $\oint_C (M dx + N dy + P dz)$.

If S is an oriented surface in space with unit normal vector field $\vec{N}$ to the surface bounded by C a closed curve compatible with S, then

$$\boxed{\oint_C \vec{F} \cdot d\vec{R} = \oint_C (M dx + N dy + P dz) = \iint_S (\text{curl}\vec{F}) \cdot \vec{N} dS}$$



Divergence Theorem in Space can be used to evaluate flux integrals $\iint \vec{F} \cdot N dS$ where S encloses a solid region.

Let S be orientable surface of a solid region D in space with unit outward normal vector field $\vec{N}$ on the surface S, then

$$\boxed{\iint_S \vec{F} \cdot \vec{N}\, dS = \iiint_D \text{div}\vec{F}\, dV}$$