Review for Exam # 1, Math 2450
Chapters 9.1-9.7,10.1,10.2,10.4,11.1-11.3

Review: Homework 1-4, Webwork 1-3, and Quiz 1.
Topics: **Dot product (scalar product, inner product):** $\vec{v}\cdot\vec{w} =< v_1, v_2, v_3 > \cdot < w_1, w_2, w_3 >= v_1w_1 + v_2w_2 + v_3w_3 = \|\vec{v}\|\|\vec{w}\|\cos(\theta)$, $\|\vec{v}\| = \sqrt{\vec{v}\cdot\vec{v}} = \sqrt{v_1^2 + v_2^2 + v_3^2}$.

Cross product (vector product, outer product): $\vec{v}\times\vec{w} = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ v_1 & v_2 & v_3 \\ w_1 & w_2 & w_3 \end{vmatrix}$.

$\|\vec{v}\times\vec{w}\| = \|\vec{v}\|\|\vec{w}\|\sin(\theta)$. Two nonzero vectors are orthogonal iff their dot product is zero. The cross product of two nonparallel vectors is orthogonal to both of the vectors. Area of a triangle determined by $\vec{v}$ and $\vec{w}$ is $(1/2)\|\vec{v}\times\vec{w}\|$. Scalar triple product can be used to find the volume of the parallelepiped determined by $\vec{u}$, $\vec{v}$ and $\vec{w}$, $|(\vec{u}\times\vec{v})\cdot\vec{w}|$. Vector projection and scalar projection. Work performed as an object moves along a line with displacement $\vec{PQ}$ against a force $\vec{F}$ is $W = \vec{F}\cdot\vec{PQ} = \|\vec{F}\|\|\vec{PQ}\|\cos(\theta)$

Lines in space: Vector $\vec{v} =< a, b, c >$ is parallel to a line and the line passes through $P_0(x_0, y_0, z_0)$. An equation for the line in **parametric form:**

$$\ell : \quad x = x_0 + at,\ y = y_0 + bt,\ z = z_0 + ct, \tag{1}$$

in symmetric form: $\dfrac{x - x_0}{a} = \dfrac{y - y_0}{b} = \dfrac{z - z_0}{c}$ (only if $a, b, c \neq 0$). Determine whether two lines are skew, parallel, coincident, or intersect in a single point.

Planes in space: Vector $\vec{N} =< A, B, C >$ is normal to the plane that passes through $P_0(x_0, y_0, z_0)$, then an equation for the plane in **point-normal form:** $A(x - x_0) + B(y - y_0) + C(z - z_0) = 0$ or in **standard form:**

$$Ax + By + Cz + D = 0. \tag{2}$$

Distance from a point $P_1(x_1, y_1, z_1)$ to a line with equation (1), p. 733: $d_{P_1,\ell} = \dfrac{||\vec{v}\times\vec{P_0P_1}\|}{||\vec{v}\|}$

Distance from a point $P_1(x_1, y_1, z_1)$ to a plane in standard form (2), p. 731: $d_{P_1,plane} = \dfrac{|Ax_1 + By_1 + Cz_1 + D|}{\sqrt{A^2 + B^2 + C^2}}$

Write an equation for a plane containing three points.

Quadric surfaces: cone, sphere, ellipsoid, paraboloid, hyperbolic paraboloid, hyperboloid of one sheet and hyperboloid of two sheets, see Table 9.2 p. 736.

Vector-valued functions: $\vec{F}(t) = x(t)\vec{i} + y(t)\vec{j} + z(t)\vec{k}$ whose graph is a curve in space. Graph the curve traced by $\vec{F}(t)$ by writing the equation in rectangular form.

$$\text{Position vector}: \ \vec{R}(t) = x(t)\vec{i} + y(t)\vec{j} + z(t)\vec{k}. \tag{3}$$

$$\text{Velocity vector}: \ \vec{V}(t) = \vec{R}'(t) = x'(t)\vec{i} + y'(t)\vec{j} + z'(t)\vec{k}.$$

$$\text{Acceleration vector}: \vec{A}(t) = \vec{R}''(t) = x''(t)\vec{i} + y''(t)\vec{j} + z''(t)\vec{k}.$$

Speed: $\|\vec{V}(t)\| = \|\vec{R}'(t)\| = \dfrac{ds}{dt}$, direction of motion: $\dfrac{\vec{V}(t)}{\|\vec{V}(t)\|}$. If $\vec{V}(t_0) \neq \vec{0}$, then $\vec{V}(t_0)$ is tangent to the graph of $\vec{R}(t)$ at $t = t_0$. Integration of $\vec{F}(t) = x(t)\vec{i} + y(t)\vec{j} + z(t)\vec{k}$:

$$\int \vec{F}(t)\,dt = \left(\int x(t)\,dt + c_1\right)\vec{i} + \left(\int y(t)\,dt + c_2\right)\vec{j} + \left(\int z(t)\,dt + c_3\right)\vec{k}.$$

Unit Tangent vector: $\vec{T}(t) = \dfrac{\vec{R}'(t)}{\|\vec{R}'(t)\|}$. **Principal Unit Normal vector:** $\vec{N}(t) = \dfrac{\vec{T}'(t)}{\|\vec{T}'(t)\|}$. Vector $\vec{T}(t)$ is tangent to the curve and points in the direction of increasing t, $\vec{N}(t)$ points toward the concave side of the curve, $\vec{T}(t)\cdot\vec{N}(t) = 0$. Given $\vec{T}(t) =< a(t), b(t) >$ in the x-y plane, then $\vec{N}(t) =< -b(t), a(t) >$ or $\vec{N}(t) =< b(t), -a(t) >$.

Given the position vector $\vec{R}(t)$ in (3), the arclength from t_0 to t_1 is $s = \displaystyle\int_{t_0}^{t_1} \sqrt{\left(\frac{dx}{dt}\right)^2 + \left(\frac{dy}{dt}\right)^2 + \left(\frac{dz}{dt}\right)^2}\,dt$.

Curvature: $\kappa = \left\|\dfrac{d\vec{T}}{ds}\right\| = \dfrac{\|\vec{T}'(t)\|}{\|\vec{R}'(t)\|} = \dfrac{\|\vec{R}'(t)\times\vec{R}''(t)\|}{\|\vec{R}'(t)\|^3}$. Plane curve: $\kappa = \dfrac{|f''(x)|}{[1 + (f'(x))^2]^{3/2}}$. The radius of curvature is $r = 1/\kappa$. A straight line has curvature $\kappa = 0$. Osculating circle is the circle tangent to the curve on the concave side having the same curvature and radius equal to $r = 1/\kappa$.

Real-valued functions of several variables: $z = f(x, y) =$ surface in space $\mathbb{R}^3$; $w = g(x, y, z) =$hypersurface in $\mathbb{R}^4$. $f(x, y) = c =$ constant $=$ level curve in the x-y plane. $g(x, y, z) = c =$ constant $=$ level surface in x-y-z space. Sketch level curves and level surfaces. Domain and Range of functions. **Limits:** $\lim_{(x,y)\to(x_0,y_0)} f(x, y)$. Show a limit does not exist. Determine values (x, y) where $z = f(x, y)$ is continuous or discontinuous.

Partial differentiation: First-order partial derivatives: $f_x = \dfrac{\partial f}{\partial x}$, $f_y = \dfrac{\partial f}{\partial y}$, Second-order partial derivatives: $f_{xx} = \dfrac{\partial^2 f}{\partial x^2}$, $f_{yy} = \dfrac{\partial^2 f}{\partial x^2}$, $f_{xy} = \dfrac{\partial^2 f}{\partial y \partial x}$, $f_{yx} = \dfrac{\partial^2 f}{\partial x \partial y}$, etc. First-order partial derivatives $f_x(x, y)$ and $f_y(x, y)$ are slopes of tangent lines at a point on the surface $z = f(x, y)$ in the direction of positive x axis or positive y axis, respectively.

Supplementary Problems: Chapter 9, pp. 744-746 # 5-23 (odd),27,29, 32,47,55; Chapter 10, pp. 806-807 # 1,3,7,9,17,19,21,27,29,31,35,37,41,43,45,47,49&51(curvature),63,69; Chapter 11, pp. 909-910 # 1,5,7,9,11,15,17, 19,32,33,34.

Some Review Problems

1. Given the vectors $\vec{u} =< -2, 1, 3 >$, $\vec{v} =< 1, 1, 2 >$ and $\vec{w} =< 0, 1, 3 >$, compute
   (a) the angle between $\vec{u}$ and $\vec{v}$.
   (b) any unit vector perpendicular to $\vec{w}$.
   (c) the volume of the parallelepiped determined by all three vectors.
   (d) equation for a plane in standard form that is parallel to $\vec{u}$ and $\vec{w}$ and contains the point $(3, 3, -2)$.
2. (a) Identify and sketch the two surfaces: $z = f(x, y) = y - 2x + 4$, $z = f(x, y) = \sqrt{36 - x^2 - 9y^2}$.
   (b) Sketch the two level curves of $f(x, y) = 2^{1-x^2+y^2}$ when $f(x, y) = 1$ and $f(x, y) = 4$.
3. Find the distance from the point $P(2, 3, 4)$ to the plane $x + y + 2z = 1$.
4. (a) Determine whether the two lines are parallel, skew or intersect. If they intersect, find the point(s) of intersection. $x = 2t$, $y = 1 - 3t$, $z = 2$ and $x = 1 - t$, $y = 2 - t$ and $z = 1 + t$.
   (b) Show that the two planes are perpendicular: $z = x + 2y - 2$ and $2x + 2z = 3$.
   (c) Write an equation for a line in parametric form that is perpendicular to the plane $z = 2x - 5y + 11$ and contains the point (3,3,$-$2).
5. Find the position and acceleration vectors and speed, $\vec{R}(t)$, $\vec{A}(t)$, $\|\vec{V}(t)\|$ given $\vec{V}(t) = \dfrac{3}{t}\vec{i} + 4e^{4t}\vec{k}$ and $\vec{R}(1) = \vec{j}$.
6. Find the unit tangent vector $\vec{T}(t)$ and principal unit normal vector $\vec{N}(t)$ for the curve given by $\vec{R}(t) =< t, t^3 >$. Sketch the curve. What is curvature at (1,1)? Radius of curvature at (1,1)? Equation for the osculating circle at (1,1)?
7. Find the length of the curve traced by the vector $\vec{R}(t) = e^t\vec{i} + e^t \cos(\pi t)\vec{j} + e^t \sin(\pi t)\vec{k}$ from $t = 0$ to $t = 2$.
8. (a) Find the limit: $\displaystyle\lim_{(x,y)\to(2,1)} \frac{x^4 - 16y^4}{x^2 - 4y^2}$. (b) Show the limit does not exist: $\displaystyle\lim_{(x,y)\to(0,0)} \frac{3x^2y^2}{x^4 + y^4}$.
9. Let $f(x, y) = e^{xy} \cos(y)$. Compute the first-order and second-order partial derivatives: f_x, f_y, f_{xx}, f_{xy}, f_{yy}.