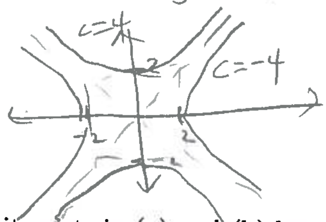
Math 2450.010, Homework # 4
Due: Thursday, September 24

1. Graph the level curves (contour plots) $f(x, y) = c$ of the function $f(x, y) = y^2 - x^2$ (hyperbolic paraboloid), when $c = 4$ and $c = -4$.

$C=4$: $y^2 - x^2 = 4 \Rightarrow \frac{y^2}{2^2} - \frac{x^2}{2^2} = 1$ hyperbola on y-axis

$C=-4$: $x^2 - y^2 = 4 \Rightarrow \frac{x^2}{2^2} - \frac{y^2}{2^2} = 1$ hyperbola on x-axis



2. The limit exists in (a) and (b) but does not exist in part (c). Find the limit in (a) and (b) and show why the limit does not exist in (c).

(a) $\lim\limits_{(x,y)\to(1,1)} \dfrac{x-y}{x^2-y^2} = \lim\limits_{(x,y)\to(1,1)} \dfrac{x-y}{(x+y)(x-y)} = \lim\limits_{(x,y)\to(1,1)} \dfrac{1}{x+y} = \dfrac{1}{2}$

(b) $\lim\limits_{(x,y)\to(2,3)} \dfrac{x-y}{x^2-y^2} = \dfrac{2-3}{4-9} = \dfrac{-1}{-5} = \dfrac{1}{5}$

(c) $\lim\limits_{(x,y)\to(0,0)} \dfrac{xy}{x^2+y^2}$ Does not exist.

Compute the limit along the line $x = 0$: $\lim\limits_{y\to 0} \dfrac{0y}{y^2} = \lim\limits_{y\to 0} \dfrac{0}{y^2} \overset{\text{L'Hopital}}{=} \lim \dfrac{0}{2y} = \lim\limits_{y\to 0} \dfrac{0}{2} = 0$

Compute the limit along the line $y = 2x$: $\lim\limits_{x\to 0} \dfrac{2x^2}{x^2+(2x)^2} = \lim\limits_{x\to 0} \dfrac{2x^2}{5x^2} = \dfrac{2}{5}$

In each case, these lines pass through the point (0,0). If the limit of $f(x, y)$ exists as $(x, y) \to (0, 0)$, then the limit should have the same value no matter how $(0, 0)$ is approached. Why does the limit not exist in part (c)? Limit is not unique. Approaching (0,0) from 2 directions leads to two different values Of $2/5$

3. The function $f(x, y) = \dfrac{x^2 - y^2}{x + y}$ is discontinuous at what values of (x, y) in the plane?

Discontinuous for $x+y=0$ or $y=-x$

Can $f(x, y)$ be re-defined where it is discontinuous to make it continuous at all values of (x, y) in the plane?

$f(x,y) = \dfrac{(x-y)(x+y)}{x+y} = x-y$ when $y=-x$

4. Compute the following partial derivatives of $f(x, y) = 4x^2y^3 - \cos(3y)$

(a) $\dfrac{\partial f}{\partial x} = 8xy^3$

(b) $\dfrac{\partial f}{\partial y} = 12x^2y^2 + 3\sin(3y)$

(c) $\dfrac{\partial^2 f}{\partial x \partial y} = 24xy^2$

(d) $\dfrac{\partial^2 f}{\partial x^2} = 8y^3$