Math 2450.H01, Homework # 4
Due: Thursday, September 24

1. The limit exists in (a) and (b) but it does not exist in part (c). Compute the limit in (a) and in (b) and show why the limit does not exist in (c).

   (a) $\lim_{(x,y)\to(1,1)} \dfrac{x-y}{x^2-y^2} =$

   (b) $\lim_{(x,y)\to(2,3)} \dfrac{x-y}{x^2-y^2} =$

   (c) $\lim_{(x,y)\to(0,0)} \dfrac{xy}{x^2+y^2}$ Does not exist.

   Compute the limit along the line $x = 0$: $\lim_{y\to 0} \dfrac{0y}{y^2} =$

   Compute the limit along the line $y = 2x$: $\lim_{x\to 0} \dfrac{2x^2}{x^2+(2x)^2} =$

   In each case, these lines pass through the point (0,0). If the limit of $f(x, y)$ exists as $(x, y) \to (0, 0)$, then the limit should have the same value no matter how $(0, 0)$ is approached. Why does the limit not exist in part (c)?

2. The function $f(x, y) = \dfrac{x^2-y^2}{x+y}$ is discontinuous at what values of (x, y) in the plane?

   Can $f(x, y)$ be re-defined where it is discontinuous to make it continuous at all values of (x, y) in the plane?

3. Compute the following partial derivatives of $f(x, y) = 4x^2y^3 - \cos(3y)$

   (a) $\dfrac{\partial f}{\partial x}$

   (b) $\dfrac{\partial f}{\partial y}$

   (c) $\dfrac{\partial^2 f}{\partial x \partial y}$

   (d) $\dfrac{\partial^2 f}{\partial x^2}$

4. Write an equation for the plane that is tangent to the surface $z = (x - 1)^2 + y^2 + 4$ at the point (1,0,4).