Math 2450.010, Homework # 3
Due: Thursday, September 17

1. Given the equation for a plane $4y - 3x + 10z = 11$.

   (a) Identify 2 points on this plane.
   Answer: There are lots of points. Give two points (x, y, z) that satisfy the plane equation. For example, $(1, 1, 1), (-11/3, 0, 0), (1, 11/4, 0), (0, 0, 11/10)$, etc.

   (b) A unit vector perpendicular to this plane.
   Answer: $< -\frac{3}{\sqrt{125}}, \frac{4}{\sqrt{125}}, \frac{10}{\sqrt{125}} >$

   (c) A unit vector parallel to this plane.
   Answer: There are lots of vectors parallel to a plane. Just find a unit vector that is perpendicular to the normal of the plane given in part (b): $< 4/5, 3/5, 0 >$ or $< 0, 10/\sqrt{116}, -4/\sqrt{116} >$ etc.

2. If $\vec{F}(t) = t^2 e^t \vec{i} - \vec{j} + 4\vec{k}$ and $\vec{G}(t) = e^{-t}\vec{i} - \vec{k}$, compute

   (a) $\vec{F}(t) \cdot \vec{G}(t)$
   Answer: The dot product is a scalar: $t^2 - 4$

   (b) Values of t where $\vec{F}(t)$ and $\vec{G}(t)$ are perpendicular.
   Answer: The dot product of part (a) is zero, $t^2 - 4 = 0$. Therefore, $t = \pm 2$.

3. Given the curve $y = 4 - x^2$ in the plane, compute

   (a) The radius of curvature at $x = 1$.
   Answer: The curvature is $\kappa = \left.\frac{|-2x|}{[1 + (-2x)^2]^{3/2}}\right|_{x=1} = \frac{2}{5^{3/2}}$. But we need the radius of curvature $r = \frac{1}{\kappa} = \frac{5^{3/2}}{2}$

   (b) The equation for the "osculating circle" at $x = 1$.
   Answer: We need to find the center of the circle (x_0, y_0) because we found the radius in part (a). The osculating circle is tangent to the parabola at (1,3) and has radius $r = 5^{3/2}/2$ in the direction of the principal unit normal to the parabola, concave side of the parabola. Therefore, to find the coordinates of the center add the vector $< 1, 3 >$ to the normal vector of length r, $r\vec{N}$ when $x = 1$. We need to use the parametric form: $\vec{R}(t) =< t, 4 - t^2 >$ and $\vec{R}'(t) =< 1, -2t >$, so $\vec{T}(t) =< 1/\sqrt{1 + 4t^2}, -2t/\sqrt{1 + 4t^2} >$. There are two normal vectors to $\vec{T}(t)$ in the plane. We want the normal that is on the concave side of the parabola at (1,3). That is, $\vec{N}(t) =< -2t/\sqrt{1 + 4t^2}, -1/\sqrt{1 + 4t^2} >$. To find the center of the osculating circle $t = x = 1$: $< x_0, y_0 >=< 1, 3 > +(5^{3/2}/2) < -2/\sqrt{5}, -1/\sqrt{5} >=< -4, 1/2 >$.

$$(x + 4)^2 + (y - 1/2)^2 = \frac{125}{4}$$