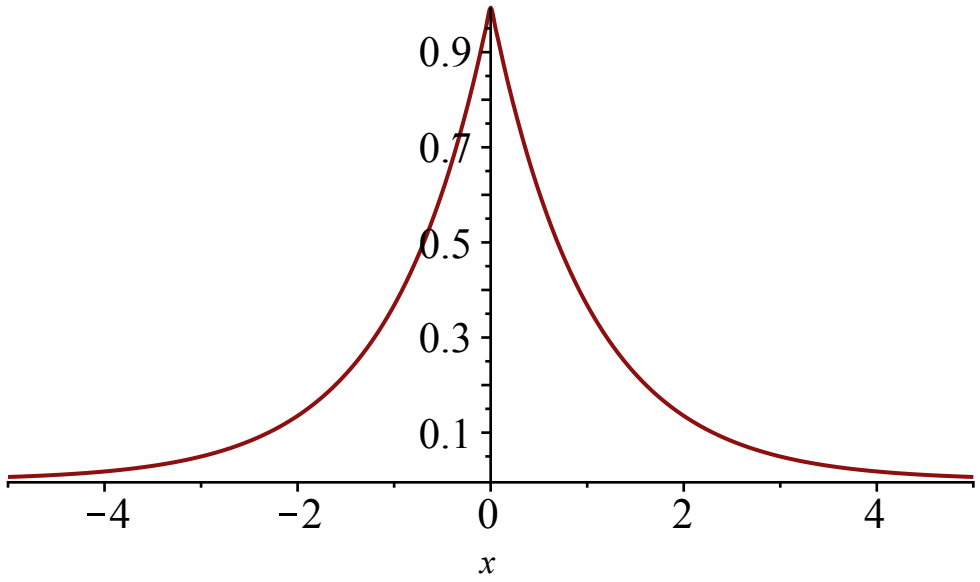
Fourier Tranforms
Solution of Heat Equation on -infinity<x<infinity
ut=uxx
u(x,0)=exp(-|x|)

```
with(plots):
u0(x) := 1/Pi·Int(cos(a·x)/(1+a^2), a=-100..100); u1(x) := 1/Pi·Int(exp(-a^2)·cos(a·x)/(1+a^2), a=-100..100);
```

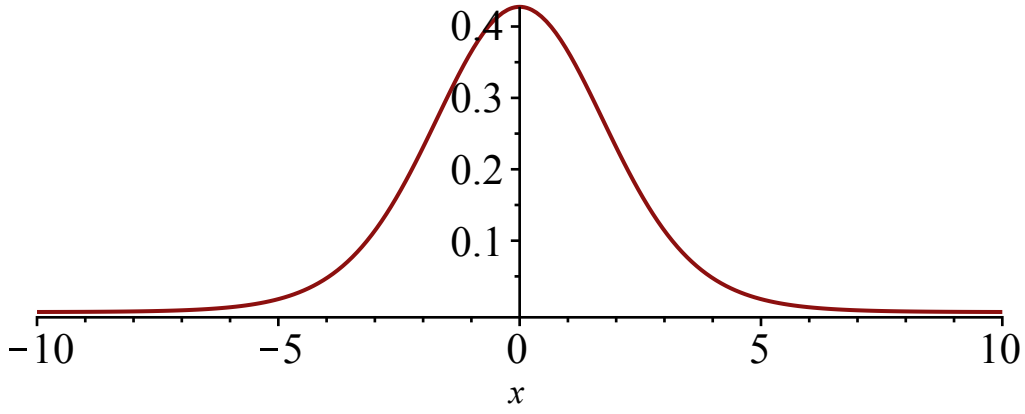
$$u0 := x \to \frac{\int_{-100}^{100} \frac{\cos(a\,x)}{a^2+1}\,da}{\pi}$$

$$u1 := x \to \frac{\int_{-100}^{100} \frac{e^{-a^2}\cos(a\,x)}{a^2+1}\,da}{\pi} \qquad \textbf{(1)}$$

```
with(plots): plot(evalf(u(x,0)), x=-5..5);
```




```
plot(evalf(u(x,1)), x=-10..10);
```



Wave Equation on 0<x<infinity
utt=uxx
u(0,t)=0
u(x,0)=xexp(-x), ut(x,0)=0

$$u0(x) := \frac{4}{\mathrm{Pi}} \cdot Int\left(\frac{\mathrm{alpha} \cdot \sin(\mathrm{alpha} \cdot x)}{(1 + \mathrm{alpha}^2)^2}, \mathrm{alpha} = 0 \ldots 1000 \right); u1(x) := \frac{4}{\mathrm{Pi}} \cdot Int\left(\frac{\mathrm{alpha} \cdot \cos(\mathrm{alpha}) \cdot \sin(\mathrm{alpha} \cdot x)}{(1 + \mathrm{alpha}^2)^2}, \mathrm{alpha} = 0 \ldots 1000 \right); u2(x) := \frac{4}{\mathrm{Pi}} \cdot Int\left(\frac{\mathrm{alpha} \cdot \cos(\mathrm{alpha} \cdot 2) \cdot \sin(\mathrm{alpha} \cdot x)}{(1 + \mathrm{alpha}^2)^2}, \mathrm{alpha} = 0 \ldots 1000 \right); u12(x) := \frac{4}{\mathrm{Pi}} \cdot Int\left(\frac{\mathrm{alpha} \cdot \cos(12 \cdot \mathrm{alpha}) \cdot \sin(\mathrm{alpha} \cdot x)}{(1 + \mathrm{alpha}^2)^2}, \mathrm{alpha} = 0 \ldots 1000 \right);$$

$$u0 := x \to \frac{4\left(\int_0^{1000} \frac{\alpha \sin(\alpha x)}{(\alpha^2 + 1)^2} \, d\alpha \right)}{\pi}$$

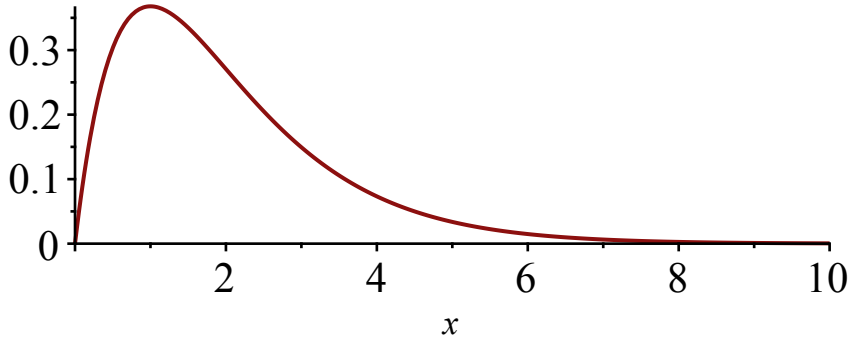
$$u1 := x \to \frac{4\left(\int_0^{1000} \frac{\alpha \cos(\alpha) \sin(\alpha x)}{(\alpha^2 + 1)^2} \, d\alpha \right)}{\pi}$$

$$u2 := x \to \frac{4\left(\int_0^{1000} \frac{\alpha \cos(2\alpha) \sin(\alpha x)}{(\alpha^2 + 1)^2} \, d\alpha \right)}{\pi}$$

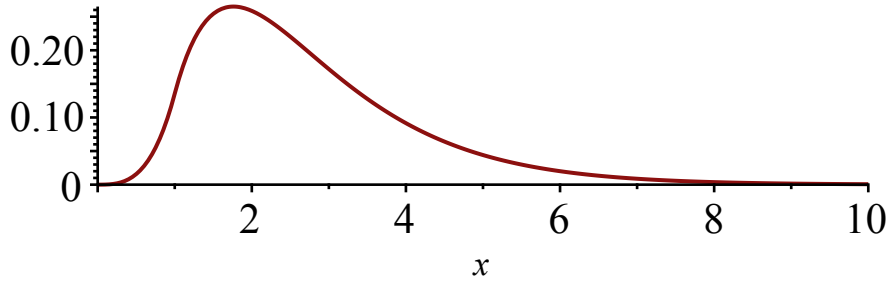
$$u12 := x \to \frac{4\left(\int_0^{1000} \frac{\alpha \cos(12\alpha) \sin(\alpha x)}{(\alpha^2 + 1)^2} \, d\alpha \right)}{\pi}$$

(2)

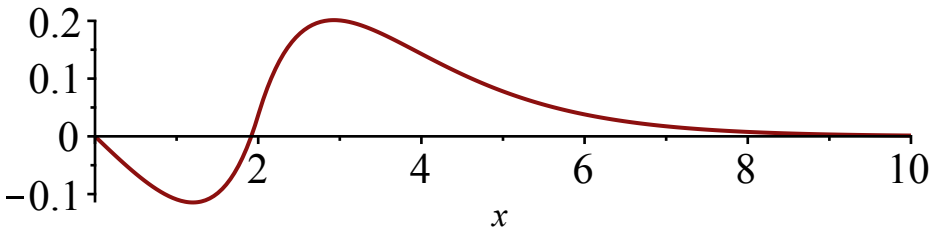
$$plot(evalf(u0(x))), x = 0 \ldots 10);$$




```
plot(evalf(u1(x)), x = 0 ..10);
```




```
plot(evalf(u2(x)), x = 0 ..10);
```




```
plot(evalf(u12(x)), x = 0 ..20);
```