Math 2450.010, Quiz 3
Fall 2015

Show all of your work. Circle your answers. Name Answers
No calculators. Turn off all electronic devices.

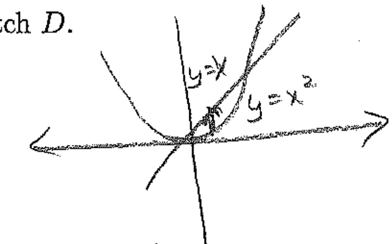
1. (20 pts) Evaluate the integrals.

(a) $\int_0^1 \int_1^3 x^2 y\,dx\,dy$ $= \int_0^1 \frac{x^3}{3}\Big|_1^3 y\,dy = \int_0^1 \left(9-\frac{1}{3}\right) y\,dy = \int_0^1 \frac{26}{3} y\,dy$

$$= \frac{26}{3}\frac{y^2}{2}\Big|_0^1 = \frac{26}{6} = \boxed{\frac{13}{3}}$$

or $\int_1^3 x^2\,dx \int_0^1 y\,dy = \frac{x^3}{3}\Big|_1^3 \frac{y^2}{2}\Big|_0^1 = \left(9-\frac{1}{3}\right)\left(\frac{1}{2}\right) = \boxed{\frac{13}{3}}$

(b) $\iint_D xy\,dy\,dx$, where D is the region bounded by $y = x^2$ and $y = x$. Sketch D.



$$\int_0^1\int_{x^2}^{x} xy\,dy\,dx = \int_0^1 x\frac{y^2}{2}\Big|_{y=x^2}^{y=x} dx$$

$$= \int_0^1 \left(\frac{x^3}{2} - \frac{x^5}{2}\right) dx = \frac{x^4}{8} - \frac{x^6}{12}\Big|_0^1$$

$$= \frac{1}{8} - \frac{1}{12} = \frac{3-2}{24} = \boxed{\frac{1}{24}}$$

(c) $\iiint_D e^z\,dV$, where D is the region described by the inequalities, $0 \le x \le 1$, $0 \le y \le x$, and $0 \le z \le x + y$.

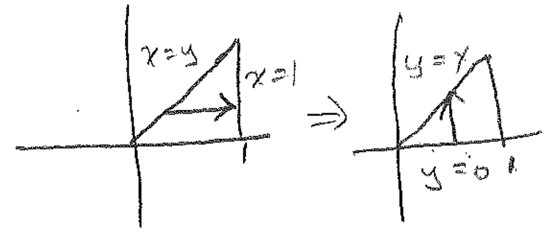
$$\int_0^1\int_0^x\int_0^{x+y} e^z\,dz\,dy\,dx = \int_0^1\int_0^x e^z\Big|_0^{x+y} dy\,dx = \int_0^1\int_0^x \big(\underbrace{e^{x+y} - e^0}_{e^{x+y}-1}\big)\,dy\,dx$$

$$= \int_0^1 e^{x+y} - y\Big|_{y=0}^{y=x} dx = \int_0^1 \left(e^{2x} - x - e^x\right) dx$$

$$= \frac{e^{2x}}{2} - \frac{x^2}{2} - e^x\Big|_0^1 = \left(\frac{e^2}{2} - \frac{1}{2} - e\right) - \left(\frac{1}{2} - 1\right) = \boxed{\frac{e^2}{2} - e}$$

2. (10 pts) Change the order of integration, then evaluate the integral $\int_0^1 \int_y^1 \sin(x^2)\, dx\, dy = \int_A^B \int_C^D \sin(x^2)\, dy\, dx$

$$\int_0^1 \int_0^x \sin(x^2)\, dy\, dx$$



$$= \int_0^1 y \sin(x^2) \Big|_{y=0}^{y=x} dx$$

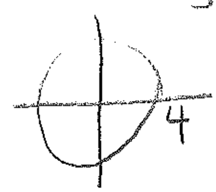
$$= \int_0^1 x \sin(x^2)\, dx = -\frac{1}{2}\cos(x^2)\Big|_0^1 = -\frac{1}{2}\cos(1) + \frac{1}{2}\cos(0)$$

$$= \boxed{\frac{1}{2} - \frac{1}{2}\cos(1)}$$

3. (10 pts) Compute the <u>surface area</u> of the plane $2x - 5y - z = 17$ that lies inside the cylinder $x^2 + y^2 = 16$.

$z = 2x - 5y - 17$ $\quad f_x = 2$ $\quad f_y = -5$

$$S = \int_0^{2\pi} \int_0^4 \sqrt{(2)^2 + (-5)^2 + 1}\; r\, dr\, d\theta$$



$$= \int_0^{2\pi} \int_0^4 \sqrt{30}\; r\, dr\, d\theta$$

$$= \sqrt{30}\; \frac{r^2}{2}\Big|_0^4 \cdot 2\pi = \boxed{16\pi\sqrt{30}}$$

4. (10 pts) Evaluate $\iiint_D \sqrt{x^2 + y^2 + z^2}\, dx\, dy\, dz$, where D is defined by $x^2 + y^2 + z^2 \le 2$.

$$\int_0^{2\pi} \int_0^{\pi} \int_0^{\sqrt{2}} \rho\, \rho^2 \sin\phi\, d\rho\, d\phi\, d\theta = \int_0^{2\pi} \int_0^{\pi} \underbrace{\frac{\rho^4}{4}\Big|_0^{\sqrt{2}}}_{1} \sin\phi\, d\phi\, d\theta$$

$$= \int_0^{2\pi} -\cos\phi \Big|_0^{\pi} d\theta = \int_0^{2\pi} \underbrace{-\cos(\pi) + \cos(0)}_{2}\, d\theta$$

$$= \boxed{4\pi}$$