Answers Exam #1

1. (a) False
   (b) False

2. (b)
   (e)

3. (a) $24t\cot(t)$
   (b) For $t \geq 0$, $\langle 4t, 5t\cos(t), -5t\sin(t) - 4t\rangle$
   (c) $3t\sin(t) = 0 \Rightarrow t = n\pi,\ n = 0, \pm 1, \pm 2, \dots$
   (d) $\lim_{t\to 0} \frac{-18t}{\sin(t)} = \lim_{t\to 0} \frac{-18}{\cos(t)} = -18$

4. $$S = \int_0^3 \|R'(t)\|\,dt$$

$$= \int_0^3 \sqrt{\underbrace{(2e^{2t})^2}_{4e^{4t}} + \underbrace{(2e^{2t}\cos(t) - e^{2t}\sin(t))^2}_{e^{4t}(4\cos^2(t) - 4\cos(t)\sin(t) + \sin^2(t))} + \underbrace{(2e^{2t}\sin(t) + e^{2t}\cos(t))^2}_{e^{4t}(4\sin^2(t) + 4\sin(t)\cos(t) + \cos^2(t))}}\,dt$$

$$= \int_0^3 \sqrt{4e^{4t} + e^{4t}(4\cos^2(t) + 4\sin^2(t)) + e^{4t}(\sin^2(t) + \cos^2(t))}\,dt$$

$$= \int_0^3 \sqrt{e^{4t}(4+4+1)}\,dt = 3\int_0^3 e^{2t}\,dt = 3\frac{e^{2t}}{2}\Big|_0^3 = \frac{3}{2}(e^6 - e^0) = \frac{3}{2}(e^6 - 1)$$

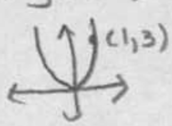
5. $\vec{PQ} \times \vec{PR} = \langle -10, 4, 3\rangle \qquad 10x - 4y - 3z - 11 = 0$

6. $\vec{A}(t) = \langle 2e^t, -1, \frac{2}{t^2}\rangle \qquad \vec{R}(t) = \langle \frac{e^{2t}}{2} - \frac{e^2}{2}, -\frac{t^2}{2} + \frac{1}{2}, -2\ln|t| + 1\rangle$

7. $$\vec{T}(t) = \frac{\vec{R}'(t)}{\|\vec{R}'(t)\|} = \langle \tfrac{3}{5}\cos(3t), \tfrac{4}{5}, -\tfrac{3}{5}\sin(3t)\rangle$$

$$\vec{N}(t) = \frac{\vec{T}'(t)}{\|\vec{T}'(t)\|} = \langle -\sin(3t), 0, -\cos(3t)\rangle$$

8. (a) $y = 3x^2$



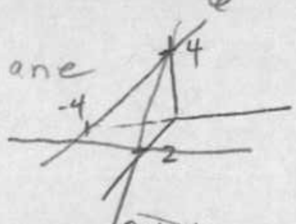
(b) $$\kappa = \frac{\|\vec{R}' \times \vec{R}''\|}{\|\vec{R}'\|^3} = \frac{\|6\vec{k}\|}{(1^2 + 36t^2)^{3/2}}\Bigg|_{t=1} = \frac{6}{37\sqrt{37}}$$

(c) $$r = \frac{1}{\kappa} = \frac{37\sqrt{37}}{6}$$

$$\vec{T} = \langle \frac{1}{\sqrt{1+36t^2}}, \frac{6t}{\sqrt{1+36t^2}}\rangle, \quad \vec{N} = \langle \frac{-6t}{\sqrt{1+36t^2}}, \frac{1}{\sqrt{1+36t^2}}\rangle$$

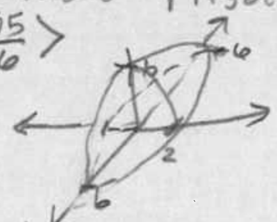
(d) $$\langle 1, 3\rangle + \frac{37\sqrt{37}}{6}\langle \frac{-6}{\sqrt{37}}, \frac{1}{\sqrt{37}}\rangle = \langle -37, \frac{55}{6}\rangle$$

9. Plane



half ellipsoid

$$\frac{z^2}{36} + \frac{x^2}{36} + \frac{y^2}{4} = 1$$



10. (a) $$\lim_{(x,y)\to(1,2)} \frac{2x-y}{(4x^2+y^2)(2x-y)(2x+y)} = \lim_{(x,y)\to(1,2)} \frac{1}{(4x^2+y^2)(2x+y)} = \frac{1}{8(4)} = \frac{1}{32}$$

(b) $y = x \quad \lim_{x\to 0} \frac{10x^2}{2x^2} = 5$

$y = 2x \quad \lim_{x\to 0} \frac{20x^2}{5x^2} = 4$

$5 \neq 4$ The limit is not unique.

11. (a) $z_x = y\cos(xy) + 10xy \qquad z_{yx} = \cos(xy) - xy\sin(xy) + 10x$

$z_y = x\cos(xy) + 5x^2 \qquad z_{yy} = -x^2\sin(xy)$

12. (a) $z - z_0 = f_x(x_0, y_0)(x - x_0) + f_y(x_0, y_0)(y - y_0)$

$z - 5 = 18(x-1) + 0(y-0)$

$z - 5 = 18x - 18 \Rightarrow z = 18x - 13$

(b) $5 = 9x^2 + y^2 - 4$

$1 = \frac{x^2}{1^2} + \frac{y^2}{3^2}$ ellipse