Answers Exam #1

1. (a) False – cross product is <u>not</u> associative! $(\vec{u}\times\vec{v})\times\vec{w} \neq \vec{u}\times(\vec{v}\times\vec{w})$
   (b) True – dot product is commutative: $\vec{a}\cdot\vec{b} = \vec{b}\cdot\vec{a}$

2. (c) $6x+3y+2z=6$
   (a) $\langle 1+2t, 1-t, 1-t\rangle$
   (e) ellipsoid with center $(1,-4,0)$: $4(x-1)^2+(y+4)^2+z^2=16$

$$\frac{(x-1)^2}{4}+\frac{(y+4)^2}{16}+\frac{z^2}{16}=1$$

3.
$$S=\int_0^2 \|\vec{R}'(t)\|\,dt=\int_0^2 \sqrt{e^{2t}\left[1+\cos^2(t)-\cancel{2\cos(t)\sin(t)}+\sin^2(t)+\sin^2(t)+\cancel{2\cos(t)\sin(t)}+\cos^2(t)\right]}\,dt$$
$$=\int_0^2 \sqrt{e^{2t}(1+1+1)}\,dt=\int_0^2 e^t\sqrt{3}\,dt$$
$$=e^t\sqrt{3}\Big|_0^2=\sqrt{3}(e^2-e^0)=\sqrt{3}(e^2-1)$$

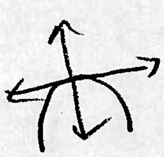
4. (a) $\langle 1,0,0\rangle$ or $\langle 0,-\cos(t),\sin(t)\rangle$ $\vec{G}(t)\cdot\langle 1,0,0\rangle=0$ or $\vec{G}(t)\cdot\langle 0,-\cos(t),\sin(t)\rangle=0$
   (b) $6t^2$
   (c) For $t\geq 0$, $5t\langle 0,\sin(t),\cos(t)\rangle+\langle 3t,0,-4t\rangle$
   $=\langle 3t, 5t\sin(t), 5t\cos(t)-4t\rangle$

5. $\vec{PQ}\times\vec{PR}=\langle 3,-9,+9\rangle$ $3x-9y+9z+D=0$, $D=-12$
   $\boxed{x-3y+3z-4=0}$

6. $\vec{V}(t)=\langle \frac{1}{2}t^2+1, -t, e^{2t}-1\rangle$ $\vec{R}(t)=\langle \frac{t^3}{6}+t, -\frac{t^2}{2}, \frac{1}{2}e^{2t}-t+\frac{1}{2}\rangle$

7. $\vec{T}(t)=\dfrac{\vec{R}'(t)}{\|\vec{R}'(t)\|}=\langle \frac{3}{5}\cos(3t), \frac{4}{5}, -\frac{3}{5}\sin(3t)\rangle$
   $\vec{N}(t)=\dfrac{\vec{T}'(t)}{\|\vec{T}'(t)\|}=\langle -\sin(3t), 0, -\cos(3t)\rangle$

8. (a) $t=x$, $\underline{y=-x^2}$



   (b) $\kappa=\dfrac{\|\vec{R}'\times\vec{R}''\|}{\|R'\|^3}=\dfrac{2}{\left(\sqrt{1+4t^2}\right)^3}\Bigg|_{t=1}=\dfrac{2}{5\sqrt{5}}$
   (c) $r=\dfrac{5\sqrt{5}}{2}$ (d) $\vec{T}=\langle \frac{1}{\sqrt{1+4t^2}}, \frac{-2t}{\sqrt{1+4t^2}}\rangle$ $\vec{N}=\langle \frac{-2t}{\sqrt{1+4t^2}}, \frac{-1}{\sqrt{1+4t^2}}\rangle$
   $\langle 1,-1\rangle+\frac{5\sqrt{5}}{2}\langle \frac{-2}{\sqrt{5}}, \frac{-1}{\sqrt{5}}\rangle=\boxed{\langle -4, -\frac{7}{2}\rangle}$

9. (a) $\lim_{(x,y)\to(2,2)} \dfrac{\cancel{(x-y)}(x+y)(x^2+y^2)}{\cancel{x-y}}=\boxed{32}$
   factor
   (b) $y=x$ $\lim_{x\to 0}\dfrac{10x^2+x^2}{x^2+x^2}=\lim_{x\to 0}\dfrac{11x^2}{2x^2}=\dfrac{11}{2}$
   $y=2x$ $\lim_{x\to 0}\dfrac{10x^2+4x^2}{x^2+4x^2}=\lim_{x\to 0}\dfrac{14x^2}{5x^2}=\dfrac{14}{5}$
   limits are not same

10. (a) $\frac{\partial z}{\partial x} = -\sin(x^2y)\frac{\partial}{\partial x}(x^2y) + 100$

$= -2xy\sin(x^2y) + 100$

(b) $\frac{\partial z}{\partial y} = -\sin(x^2y)\frac{\partial}{\partial y}(x^2y) + 15y^2$

$= -x^2\sin(x^2y) + 15y^2$

(c) $\frac{\partial^2 z}{\partial x \partial y} = \frac{\partial}{\partial x}\left(-x^2\sin(x^2y) + 15y^2\right)$

$= -2x\sin(x^2y) - x^2\cos(x^2y)\frac{\partial}{\partial x}(x^2y)$

product rule

$= -2x\sin(x^2y) - 2x^3y\cos(x^2y)$

(d) $\frac{\partial^2 z}{\partial y^2} = \frac{\partial}{\partial y}\left(-x^2\sin(x^2y) + 15y^2\right)$

$= -x^2\cos(x^2y)\frac{\partial}{\partial y}(x^2y) + 30y$

$= -x^4\cos(x^2y) + 30y$

11. (a) $f_x(0,1) = 2x\Big|_{\substack{x=0\\y=1}} = 0$

(b) $f_y(0,1) = 8y\Big|_{\substack{x=0\\y=1}} = 8$

(c) $x^2+4y^2+3=7$

$x^2+4y^2=4$

$\frac{x^2}{2^2}+\frac{y^2}{1^2}=1$ ellipse