**ABSTRACT.** We study the degenerate parabolic equation with time-dependent flux boundary condition for generalized Forchheimer (non-Darcy) flows of slightly compressible fluids in porous media. The solution is estimated, particularly for large time, in $L^\infty$-norm, $W^{1,r}$-norm for $r \geq 1$, and $W^{2,2-\delta}$-norm for $\delta > 0$. The $L^\infty$-estimates of the solution’s time derivative are also obtained. The De Giorgi and Ladyzhenskaya-Uraltseva iteration techniques are combined with uniform Gronwall-type estimates, specific monotonicity properties and suitable parabolic Sobolev embeddings. This is joint work with Thinh Kieu and Tuoc Phan.