Optimal Boundary Control for Wave Equation

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We consider the following boundary problem for the wave equation

$$u_{tt}(x,t) - u_{xx}(x,t) = 0,\ 0 < x < l,\ 0 < t < T, \tag{1}$$

$$u_x(0,t) = \mu(t),\ u(l,t) = \nu(t),\ 0 \le t \le T, \tag{2}$$

$$u(x,0) = \varphi_1(x),\ u_t(x,0) = \psi_1(x),\ 0 \le x \le l. \tag{3}$$

The solution of the problem (1), (2), (3), i.e. the function $u(x,t)$, is supposed to be in the class of distributions $\widehat{W}_2^1$ (which is some analogue of Sobolev space W_2^1).

Further, we suppose that conditions (2) are posed so that the solution $u(x,t)$, satisfying conditions (3) in the initial moment $t = 0$ satisfies in the final moment $t = T$ the supplementary conditions

$$u(x,T) = \varphi_2(x),\ u_t(x,T) = \psi_2(x),\ 0 \le x \le l. \tag{4}$$

Thus, the question arises: how the functions $\mu(t)$ and $\nu(t)$ in (2) should be defined (i.e., what is a correlation between $\mu(t)$ and $\nu(t)$ and the given functions $\varphi_1(x)$, $\varphi_2(x)$, $\psi_1(x))$, $\psi_2(x)$) in order the integral of boundary energy

$$\int_0^T ([\mu(t)]^2 + [\nu'(t)]^2)dt$$

has the minimal (among all possible) value.