Texas Tech University. Analysis Seminars.

Analytic continuation of Laurent series to domains of minimal capacity

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ABSTRACT. Let $f(z)$ be a function defined by Laurent series

$$f(z) = \sum_{k=-\infty}^{\infty} a_k z^k, \tag{1}$$

which is analytic on the unit circle $\mathbb{T}$. By the classical theorem of Pierre Alphonse Laurent (1843), the series (1) converges on an annulus $A(r, R) = \{z : r < |z| < R\}$ with some r and R, $0 \le r < 1 < R \le \infty$, and diverges for all $z \in \overline{\mathbb{C}} \setminus \overline{A(r, R)}$. It is also well-known that often the function $f(z)$ defined by the series (1) can be analytically/meromorphycally continued to a larger domain $D \supset A(r, R)$. The primary goal of this project is to study *the largest domains* $D \subset \overline{\mathbb{C}}$, to which the function f can be extended as a single-valued meromorphic function. To clarify the term "largest", we note that the complement $\overline{\mathbb{C}} \setminus D$ consists of two parts: $E_0 = \{z \notin D : |z| \ge R\}$ and $E_1 = \{z \notin D : |z| \le r\}$. If E_0 and E_1 both are not empty, then the domain D can be considered as a field of the condenser (D, E_0, E_1) with plates E_0 and E_1. Precisely, we want to identify all domains D to which the series (1) can be continued meromorphically and such that the corresponding condensers (D, E_0, E_1) have the **minimal possible capacity**. The capacity of a condenser can be defined as the minimum of the Dirichlet integral; i.e.,

$$\mathrm{cap}(D) = \inf \int_D |\nabla u|^2 \, dA,$$

where the infimum is taken over all functions $u \in Lip(D)$ such that $u = 0$ on E_0 and $u = 1$ on E_1.

This project can be thought as an extension of recent work of Prof. H. Stahl who studied meromorphic extensions of Taylor series at $z = \infty$ to domains $D \ni \infty$ with minimal logarithmic capacity of their complement $\mathbb{C} \setminus D$.

In the first part of my talk, I will introduce our main problem and discuss several examples illustrating main ideas and concepts. Then I will present an existence theorem for extremal condensers and give a sketch of its proof. Also, a uniqueness theorem for extremal domains will be discussed. Finally, the topological structure of complementary sets (or plates) E_0 and E_1 of the condensers of minimal capacity will be discussed and analytic tools needed for their characterization will be presented.