SOLUTIONS

MATH2450. SECTION 010. FALL 2012.

MIDTERM EXAMINATION 3-A

Name (in CAPITALS): ....

Signature: .... Date: ....

READ AND FOLLOW THESE INSTRUCTIONS

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INSTRUCTIONS FOR MULTIPLE CHOICE PROBLEMS (Questions 1 – 4):

There are 4 multiple choice problems, worth 15 points each.

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INSTRUCTIONS FOR THE WORK OUT PROBLEMS (Questions 5 – 6):

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PART I. Multiple Choice Problems. (Questions 1 – 4)

1. (15 points) Evaluate the double integral

$$I = \iint_D (x + 2y)dA,$$

where D is the triangle bounded by the x-axis, y-axis and the line $y = -x + 2$.

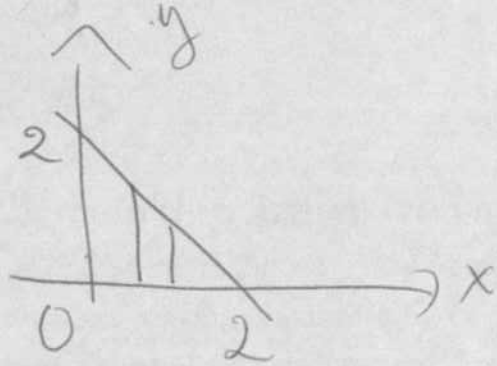
(a) -4

(b) 0

(c) 4

(d) 12

(e) 6



D: $0 \le x \le 2$

$0 \le y \le -x+2$

$$I = \int_0^2 \int_0^{-x+2} (x+2y)\,dy\,dx$$

$$= \int_0^2 x(-x+2) + y^2 \Big|_{y=0}^{y=-x+2} dx$$

$$= \int_0^2 x(-x+2) + (-x+2)^2\,dx$$

$$= \int_0^2 (-x+2)(\cancel{x-}x+2)\,dx$$

$$= \int_0^2 -2x+4\,dx = -x^2+4x\Big|_0^2$$

$$= -4+8 = 4$$

2. (15 points) Find surface area of the portion of the surface $z = 2 + x^2 + y^2$ above the disk $x^2 + y^2 \leq 9$.

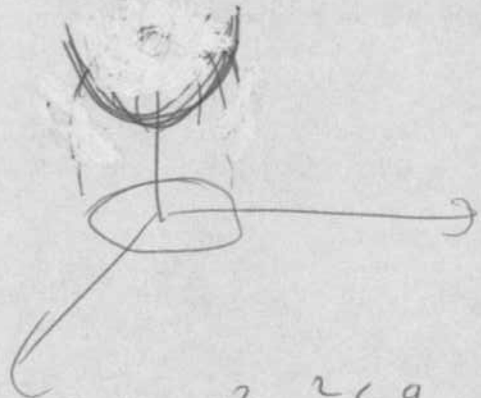
(a) $\frac{\pi}{6}(37\sqrt{37} - 1)$

(b) $2\pi(10\sqrt{10} - 1)$

(c) $2\pi\left(\frac{1}{10\sqrt{10}} - 1\right)$

(d) $\frac{4\pi}{3}(37\sqrt{37} - 1)$

(e) $\pi\left(\frac{1}{37\sqrt{37}} - 1\right)$



$D \doteq \quad x^2 + y^2 \leq 9 \quad$ radius 3

$$S = \iint_D \sqrt{1 + z_x^2 + z_y^2}\ dA$$

$z_x = 2x$
$z_y = 2y$

$$= \iint_D \sqrt{1 + 4(x^2+y^2)}\, dA$$

Use polar coords: $\quad dA = r\,dr\,d\theta$

$r^2 = x^2 + y^2.$

$D: \quad 0 \leq \theta \leq 2\pi$
$\quad\quad 0 \leq r \leq 3$

$$S = \int_0^{2\pi}\int_0^3 \sqrt{1+4r^2}\ r\ dr\ d\theta$$

$$= 2\pi\ \frac{2}{3}\cdot\frac{1}{8}\left(1+4r^2\right)^{3/2}\Big|_{r=0}^{3}$$

$$= 2\pi\cdot\frac{2}{3}\cdot\frac{1}{8}\left(37^{3/2} - 1\right)$$

$$= \frac{\pi}{6}\left(37\sqrt{37} - 1\right)$$

3. (15 points) Evaluate the triple integral

$$I = \iiint_D z\,dV,$$

where D is the solid given by

$$0 \le x \le 2, \quad 0 \le y \le 3x, \quad 0 \le z \le x + y.$$

(a) 64

(b) 84

(c) $\dfrac{128}{3}$

(d) 128

(e) 42

$$I = \int_0^2 \int_0^{3x} \int_0^{x+y} z\,dz\,dy\,dx$$

$$= \int_0^2 \int_0^{3x} \frac{1}{2}(x+y)^2\,dy\,dx$$

$$= \int_0^2 \frac{1}{2} \cdot \frac{1}{3} (x+y)^3 \Big|_{y=0}^{y=3x} dx$$

$$= \int_0^2 \frac{1}{2} \cdot \frac{1}{3} \cdot \big(\underbrace{(4x)^3 - x^3}_{63x^3}\big)\,dx$$

$$= \frac{1}{2} \cdot \frac{1}{3} \cdot 63 \cdot \frac{1}{4} x^4 \Big|_0^2$$

$$= \frac{1}{2} \cdot \frac{1}{3} \cdot 63 \cdot \frac{1}{4} \cdot 16$$

$$= 2 \cdot \frac{21}{\cancel{6}} = 42$$

4. (15 points)

Find volume of a solid D given in the spherical coordinate (ρ, θ, ϕ) by

$$D:\ 0 \le \theta \le \pi/2,\ 0 \le \phi \le \pi,\ 1 \le \rho \le 3.$$

(a) $\frac{26\pi}{3}$

(b) $\frac{26\pi^2}{3}$

(c) $\frac{4\pi}{2}$

(d) $2\pi^2$

(e) 13π

$$V = \int_0^{\pi/2}\int_0^{\pi}\int_1^{3} 1\rho^2 \sin\phi \, d\rho \, d\phi \, d\theta$$

$$= \left(\int_0^{\pi/2} d\theta\right)\left(\int_0^{\pi} \sin\phi \, d\phi\right)\left(\int_1^{3} \rho^2 d\rho\right)$$

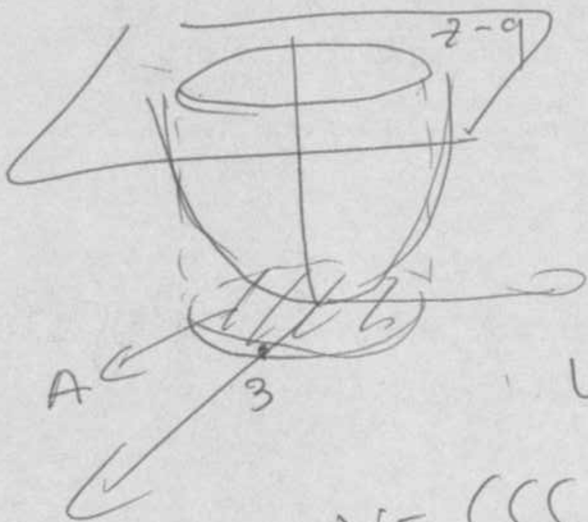
$$= \frac{\pi}{2} \cdot \left[-\cos\phi\right]_0^{\pi} \cdot \frac{1}{3}\rho^3\Big|_1^3$$

$$= \frac{\pi}{2} \cdot 2 \cdot \frac{1}{3}(27-1)$$

$$= \pi \cdot \frac{26}{3}$$

PART II. Work Out Problems. (Questions 5 - 6)

5. (20 points) Find the volume of the solid D bounded above by the plane $z = 3$ and below by the surface $3z = x^2 + y^2$. $\Leftrightarrow z = \frac{r^2}{3}$



Intersection

$9 = x^2 + y^2$

A: $x^2 + y^2 \leq 9$

radius 3.

Use cylindrical.

$$V = \iiint_D 1 \, dV$$

D: $x^2 + y^2 \leq 9$

$\frac{r^2}{3} \leq z \leq 3$

A: $0 \leq \theta \leq 2\pi$

$0 \leq r \leq 3$

$$= \int_0^{2\pi} \int_0^3 \int_{r^2/3}^{3} 1 \, dz \, r \, dr \, d\theta$$

$$= \int_0^{2\pi} \int_0^3 \left(3 - \frac{r^2}{3}\right) r \, dr \, d\theta$$

$$= 2\pi \int_0^3 3r - \frac{r^3}{3} \, dr$$

$$= 2\pi \cdot \left[\frac{3r^2}{2} - \frac{1}{3} \cdot \frac{r^4}{4} \right]_0^3$$

$$= 2\pi \left(\frac{27}{2} - \frac{27}{4} \right)$$

$$= 2\pi \cdot \frac{27}{4} = \frac{27\pi}{2}$$

6. (20 points) Let R be the square having vertices $A(1,0)$, $B(0,1)$ $C(-1,0)$ and $D(0,-1)$.

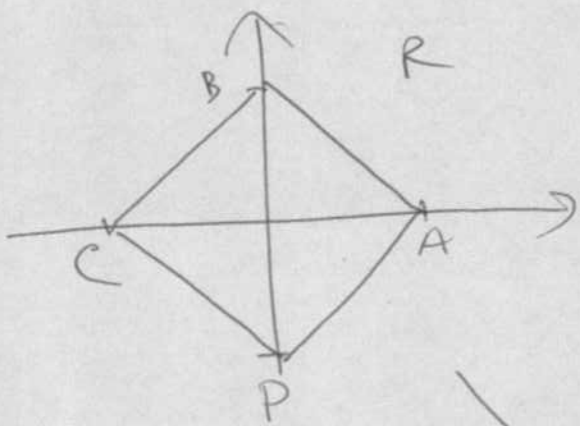
(a) Let $u = x + y$ and $v = x - y$. The region R in xy-plane is then transformed to a region R^* in uv-plane. Find and sketch the corresponding region R^* in the uv-plane.

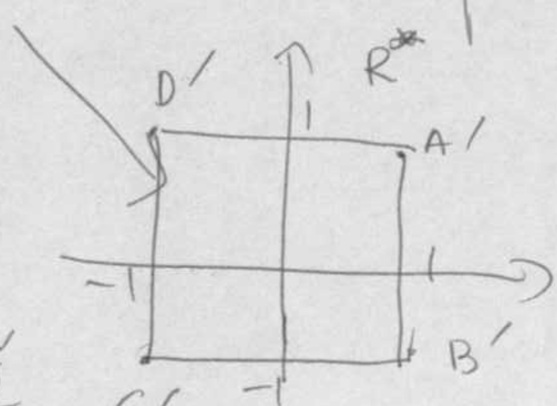
(b) Use the trasformation in part (a) to calculate the following integral

$$I = \iint_R (x+y)^2(x-y)^2 dxdy.$$

(*Hint: You can find $x = x(u,v), y = y(u,v)$ first in order to calculate the Jacobian* $\dfrac{\partial(x,y)}{\partial(u,v)}$.)

(x,y)	(u,v)
$A(1,0)$	$A'(1,1)$
$B(0,1)$	$B'(1,-1)$
$C(-1,0)$	$C'(-1,-1)$
$D(0,1)$	$D'(1,-1)$





$$\begin{cases} u = x+y \\ v = x-y \end{cases} \Rightarrow \begin{cases} 2x = u+v & x = \frac{u+v}{2} \\ 2y = u-v & y = \frac{u-v}{2} \end{cases}$$

$$\frac{\partial(x,y)}{\partial(u,v)} = \det\begin{pmatrix} \frac{1}{2} & \frac{1}{2} \\ \frac{1}{2} & -\frac{1}{2} \end{pmatrix} = -\frac{1}{4} - \frac{1}{4} = -\frac{1}{2} \qquad \left|\frac{\partial(x,y)}{\partial(u,v)}\right| = \frac{1}{2}$$

$$I = \iint_{R^*} u^2 v^2 \cdot \frac{1}{2} \cdot du dv = \int_{-1}^{1}\int_{-1}^{1} \frac{1}{2} u^2 v^2 du dv$$

$$= \frac{1}{2} \cdot \frac{2}{3} \cdot \frac{2}{3} = \frac{2}{9}$$

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1. (15 points)

Evaluate the double integral

$$I = \iint_D (x - 2y)\,dA,$$

where D is the triangle bounded by the x-axis, y-axis and the line $y = x + 2$.

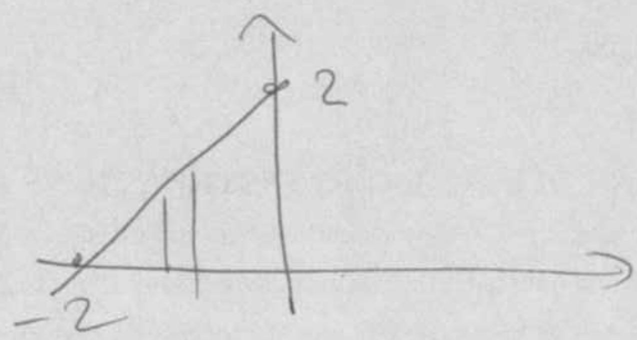
(a) 4

(b) -4

(c) 0

(d) -6

(e) 12



$$D: \quad -2 \le x \le 0$$
$$0 \le y \le x+2$$

$$I = \int_{-2}^{0} \int_{0}^{x+2} (x - 2y)\,dy\,dx$$

$$= \int_{-2}^{0} x(x+2) - (x+2)^2 \, dx$$

$$= \int_{-2}^{0} (x+2)(\cancel{x} - x - 2)\,dx$$

$$= \int_{-2}^{0} -2x - 4\,dx = -x^2 - 4x \Big|_{-2}^{0}$$

$$= +4 - 8 = -4$$

2. (15 points)

Find surface area of the portion of the surface $z = 3 + x^2 + y^2$ above the disk $x^2 + y^2 \leq 4$.

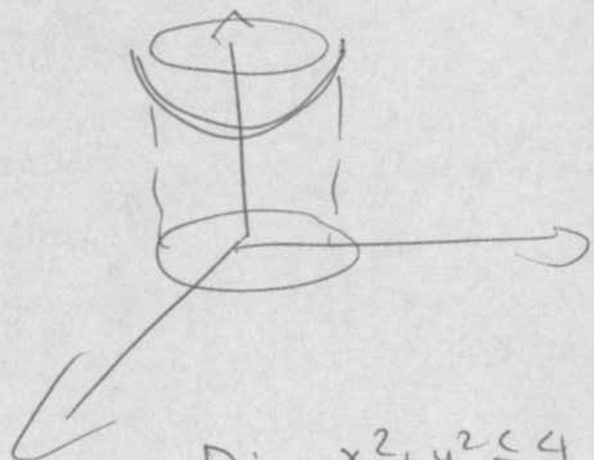
(a) $\frac{\pi}{6}(17\sqrt{17} - 1)$

(b) $\frac{4\pi}{3}(17\sqrt{17} - 1)$

(c) $2\pi(5\sqrt{5} - 1)$

(d) $2\pi\left(\frac{1}{17\sqrt{17}} - 1\right)$

(e) $\pi\left(\frac{1}{5\sqrt{5}} - 1\right)$



$D: \ x^2 + y^2 \leq 4$

radius 2.

$z_x = 2x$

$z_y = 2y$

$$S = \iint_D \sqrt{1 + z_x^2 + z_y^2}\, dA$$

$$= \iint_D \sqrt{1 + 4(x^2+y^2)}\, dA$$

Polar coords.

$D: \ 0 \leq \theta \leq 2\pi$

$0 \leq r \leq 2$

$$= \int_0^{2\pi} \int_0^2 \sqrt{1+4r^2}\, r\, dr\, d\theta$$

$$= 2\pi \cdot \frac{2}{3} \cdot \frac{1}{8} (1+4r^2)^{3/2} \Big|_{r=0}^{r=2}$$

$$= 2\pi \cdot \frac{2}{3} \cdot \frac{1}{8} \cdot \left(17^{3/2} - 1\right)$$

$$= \frac{\pi}{6}\left(17\sqrt{17} - 1\right)$$

3. (15 points)

Evaluate the triple integral

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where D is the solid given by

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(a) 243

(b) $\frac{13 \times 27}{2}$

(c) $\frac{243}{8}$

(d) $\frac{13 \times 27}{4}$

(e) 81

$$\int_0^3 \int_0^{2x} \int_0^{x+y} z \, dz\, dy\, dx$$

$$= \int_0^3 \int_0^{2x} \frac{1}{2}(x+y)^2 \, dy\, dx$$

$$= \int_0^3 \frac{1}{2} \cdot \frac{1}{3} (x+y)^3 \Big|_{y=0}^{y=2x} dy$$

$$= \int_0^3 \frac{1}{2} \cdot \frac{1}{3} \Big(\underbrace{(3x)^3 - x^3}_{26x^3} \Big) dy$$

$$= \frac{1}{2} \cdot \frac{1}{3} \cdot 26 \cdot \frac{1}{4} x^4 \Big|_0^3$$

$$= \frac{1}{\cancel{2}} \cdot \frac{1}{3} \cdot 26 \cdot \frac{1}{4} \cdot 3^4$$

$$= \frac{13}{4} \cdot 3^3 = \frac{13 \times 27}{4}$$

4. (15 points)

Find volume of a solid D given in the spherical coordinate (ρ, θ, ϕ) by

$$D:\ 0 \leq \theta \leq \pi,\ 0 \leq \phi \leq \pi/2,\ 1 \leq \rho \leq 2.$$

(a) $\frac{7\pi^2}{3}$

(b) $\frac{7\pi}{3}$

(c) $\frac{3\pi}{2}$

(d) $\frac{3\pi^2}{2}$

(e) 3π

$$\int_0^{\pi}\int_0^{\pi/2}\int_1^2 \rho^2 \sin\phi \, d\rho \, d\phi \, d\theta$$

$$= \left(\int_0^{\pi} d\theta\right)\left(\int_0^{\pi/2} \sin\phi \, d\phi\right)\left(\int_1^2 \rho^2 d\rho\right)$$

$$= \pi \cdot \left(-\cos\phi \Big|_0^{\pi/2}\right) \cdot \frac{1}{3}\rho^3 \Big|_1^2$$

$$= \pi \cdot (1) \cdot \frac{1}{3}(8-1)$$

$$= \frac{7\pi}{3}.$$

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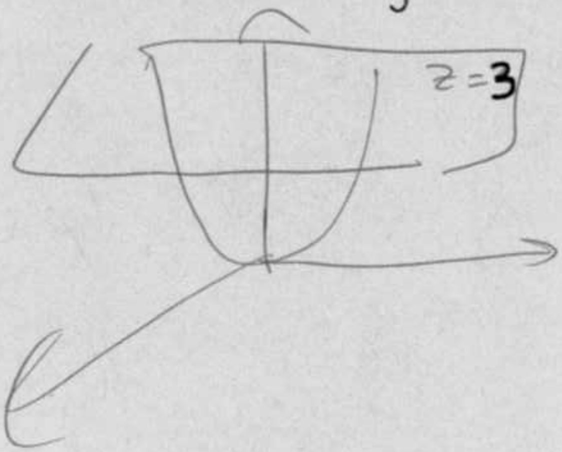

Same as 3A.

$z=3$

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