Quiz 10 (last) Fri 12/3/2010.

let $A = \begin{pmatrix} 2 & -2 \\ 2 & -3 \end{pmatrix}$

① Find eigenvalues and corresponding eigenvectors.

② Find diagonal 2×2 matrix D and non-singular 2×2 matrix X such that

$$D = X^{-1} A X.$$

③ Let $k \geq 1$ be any integer. Calculate matrix A^k.

Solution: $\mathrm{trace}(A) = -1$,

$\det A = -6 + 4 = -2$.

① characteristic poly: $p(\lambda) = \lambda^2 + \lambda - 2 = 0$

$$\lambda = 1, -2$$

$\underline{\lambda = 1}$ $\quad A - I = \begin{pmatrix} 1 & -2 \\ 2 & -4 \end{pmatrix} \to \begin{pmatrix} \boxed{1} & -2 \\ 0 & 0 \end{pmatrix}$

eigenspace $\to$ $N(A - I)$ has basis $\left\{ \begin{pmatrix} 2 \\ 1 \end{pmatrix} \right\} = \{v_1\}$

$\lambda = -2$ $\quad \lambda + 2I = \begin{pmatrix} 4 & -2 \\ 2 & -1 \end{pmatrix} \to \begin{pmatrix} 2 & -1 \\ 0 & 0 \end{pmatrix}$ $\quad x_1 = 1$, $x_2 = 2$

eigenspace $\to$ $N(A + 2I)$ has basis $\left\{ \begin{pmatrix} 1 \\ 2 \end{pmatrix} \right\} = \{v_2\}$

② $X = (v_1\ v_2) = \begin{pmatrix} 2 & 1 \\ 1 & 2 \end{pmatrix}$

$D = \begin{pmatrix} 1 & 0 \\ 0 & -2 \end{pmatrix} = X^{-1} A X.$

$\longrightarrow$ over

(3)

$$A = X D X^{-1}$$

$$A^k = X D^k X^{-1}$$

$$= \begin{pmatrix} 2 & 1 \\ 1 & 2 \end{pmatrix} \begin{pmatrix} 1 & 0 \\ 0 & (-2) \end{pmatrix}^k \begin{pmatrix} 2 & 1 \\ 1 & 2 \end{pmatrix}^{-1}$$

$$= \begin{pmatrix} 2 & 1 \\ 1 & 2 \end{pmatrix} \begin{pmatrix} 1 & 0 \\ 0 & (-2)^k \end{pmatrix} \frac{1}{3} \begin{pmatrix} 2 & -1 \\ -1 & 2 \end{pmatrix}$$