COUNTING ARITHMETICAL STRUCTURES

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ABSTRACT. Let G be a finite, simple, connected graph. An arithmetical structure on G is a pair of positive integer vectors (d, r) such that $(\mathrm{diag}(d) - A)r = 0$, where A is the adjacency matrix of G. Arithmetical structures were introduced in the context of arithmetical geometry by Lorenzini in 1989 to model intersections of curves. However, they also naturally appear in the context of algebraic graph theory as arithmetical structures generalize the notion of the Laplacian matrix of a graph. In this talk, I will present a general introduction to the topic, we will focus on the combinatorics of arithmetical structures on certain graph families such as path, cycles, complete and bident graphs. We will also present some results about the associated critical group of an arithmetical structure, i.e., the cokernel of the matrix $(\mathrm{diag}(d) - A)$. We will particularly focus on counting arithmetical structures. For paths, we will show that arithmetical structures are enumerated by the Catalan numbers, and we obtain refined enumeration results related to ballot sequences. For cycles, we prove that arithmetical structures are enumerated by the binomial coefficients $C(2n - 1, n - 1)$, and we obtain refined enumeration results related to multisets.