SYZYGIES IN AND FROM GEOMETRIC COMBINATORICS

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Abstract. In this survey talk we discuss several instances where objects and constructions from geometric/topological combinatorics in fact encode underlying *algebraic* relations among monomial and binomial ideals. In one direction, polytopal complexes which parametrize combinatorial data (homomorphism complexes, generalized permutohedra, secondary polytopes) lead to minimal cellular resolutions. In another, combinatorial moves that preserve topological properties of simplicial complexes (elementary collapses) lead to a large class of ideals with linear resolutions, generalizing Froberg's theorem for edge ideals and leading to higher-dimensional notions of chordality. In many cases the underlying ideals are well-studied in the algebraic community (stable ideals, determinantal ideals, toppling ideals) and lead to new results regarding resolutions and Betti numbers. At the same time algebraic properties of these ideals lead to new insights regarding the underlying combinatorial objects.