A NON-PARTITIONABLE COHEN-MACAULAY SIMPLICIAL COMPLEX, AND IMPLICATIONS FOR STANLEY DEPTH

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Abstract. Cohen–Macaulay simplicial complexes are those whose face-ring is a Cohen–Macaulay ring; there is also a topological characterization. A simplicial complex is "partitionable" if its partially ordered set of faces can be partitioned into certain kinds of intervals. In 1979, Richard Stanley posed the following conjecture relating these ideas, which he later described as " central combinatorial conjecture on Cohen–Macaulay complexes":

Every Cohen–Macaulay simplicial complex is partitionable

We disprove this conjecture by constructing an explicit counterexample in three dimensions. Due to a result of Herzog, Jahan and Yassemi, our construction also disproves the well-known conjecture that the Stanley depth of a monomial ideal is always at least its depth. I will describe the background of the problem and our construction, and new open questions. This is joint work with Bennet Goeckner, Carly Klivans, and Jeremy Martin.