THE ALGEBRA OF GENERALIZED BARYCENTRIC COORDINATES

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Abstract. Let $P_d$ be a convex polygon with $d$ vertices. The associated Wachspress surface $W_d$ is a fundamental object in approximation theory, defined as the image of the rational map $w_d$ from $P^2$ to $P^{d-1}$, determined by the Wachspress barycentric coordinates for $P_d$. We show $w_d$ is a regular map on a blowup $X_d$ of $P^2$, and if $d > 4$ is given by a very ample divisor on $X_d$, so has a smooth image $W_d$. We determine generators for the ideal of $W_d$, and prove that in graded lex order, the initial ideal of $I(W_d)$ is given by a Stanley-Reisner ideal. As a consequence, we show that the associated surface is arithmetically Cohen-Macaulay, of Castelnuovo-Mumford regularity two, and determine all the graded Betti numbers of $I(W_d)$. Joint work with Corey Irving, Santa Clara Univ.