Bounded Archimedean Lattice-Ordered $\mathbb{R}$-Algebras

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Abstract: The $\mathbb{R}$-algebra $C(X, \mathbb{R})$ of continuous real-valued functions on a topological space X has been well studied. In the 1930s Stone proved that the category of compact Hausdorff spaces is dually equivalent to the category $\mathcal{C}$ of such algebras. He also axiomatized these algebras as certain complete normed lattice-ordered $\mathbb{R}$-algebras. We will study a larger category $\boldsymbol{ba\ell}$, consisting of bounded Archimedean lattice-ordered $\mathbb{R}$-algebras. Each algebra in $\boldsymbol{ba\ell}$ is isomorphic to a lattice-ordered subalgebra of $C(X, \mathbb{R})$ for some compact Hausdorff space X.

In this talk we will motivate our study of the category $\boldsymbol{ba\ell}$ and give background on lattice-ordered algebras and the duality between compact Hausdorff spaces and rings of continuous functions. We will see that the category $\mathcal{C}$ can be described as the unique reflective epicomplete subcategory of $\boldsymbol{ba\ell}$. We will also discuss some other interesting subcategories of $\boldsymbol{ba\ell}$.

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