Factorizations of Algebraic Integers, Block Monoids and Additive Number Theory

Scott Chapman

Sam Houston State University
Department of Mathematics and Statistics
Huntsville, TX 77340

Abstract

Let \( \mathcal{O}_K \) be the ring of integers in a finite extension of the rationals. Fundamental problems involving the factorizations of elements in \( \mathcal{O}_K \) into irreducibles appear early in the Abstract Algebra curriculum. At a basic level, the usual technique for attacking such problems involves using the norm function. There is a much deeper connection between factorizations of elements in \( \mathcal{O}_K \) and the class group of \( \mathcal{O}_K \). We will explore this connection and show that it easily generalizes to Dedekind and Krull domains. Implicit in this discussion is the introduction of a structure known as a Block Monoid. A well-known theorem of Geroldinger constructs a monoid homomorphism from \( \mathcal{O}_K \) to an appropriately chosen Block Monoid \( B \) which preserves lengths of factorizations of elements into products of irreducibles. The analysis of the factorization properties of Block Monoids leads to the study of two well-known arithmetic constants from Additive Number Theory, the Davenport Constant and the Cross Number.

Email address: scott.chapman@shsu.edu (Scott Chapman).