Quiz 7
Mathematics 208
Spring 2006

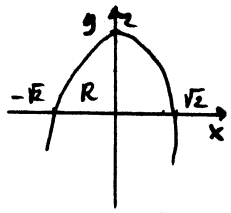
Name _____

1. Let R be the region in $\mathbb{R}^2$ bounded by the x-axis and the graph $y = 2 - x^2$. Sketch R and compute the double integral

$$\iint_R x + 1 \, dA$$

2. Sketch the tetrahedron bounded by the plane $x + 2y + z = 1$ and the xy-, xz-, and yz-planes, and compute its volume.

(1)



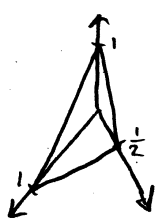
$$\iint_R x+1 \, dA = \int_{-\sqrt{2}}^{\sqrt{2}} \int_0^{2-x^2} x+1 \, dy \, dx$$

$$= \int_{-\sqrt{2}}^{\sqrt{2}} \Big| (x+1)y \Big|_0^{2-x^2} dx = \int_{-\sqrt{2}}^{\sqrt{2}} (x+1)(2-x^2) \, dx$$

$$= \int_{-\sqrt{2}}^{\sqrt{2}} -x^3 - x^2 + 2x + 2 = \Big| -\tfrac{1}{4}x^4 - \tfrac{1}{3}x^3 + x^2 + 2x \Big|_{-\sqrt{2}}^{\sqrt{2}}$$

$$= -\tfrac{1}{4}4 - \tfrac{1}{3}2\sqrt{2} + 2 + 2\sqrt{2} - \left(-\tfrac{1}{4}4 - \tfrac{1}{3}2(-\sqrt{2}) + 2 - 2\sqrt{2}\right) = 4\sqrt{2} - \tfrac{4}{3}\sqrt{2} = \underline{\underline{\tfrac{8}{3}\sqrt{2}}}$$

(2)



Set $R = \{(x,y) \in \mathbb{R}^2 \mid 0 \le x \le 1, \text{ and } 0 \le y \le \frac{1-x}{2}\}$

$$\text{Vol} = \iint_R 1 - x - 2y \, dA = \int_0^1 \int_0^{\frac{1-x}{2}} 1 - x - 2y \, dy \, dx$$

$$= \int_0^1 \Big| (1-x)y - y^2 \Big|_0^{\frac{1-x}{2}} dx = \int_0^1 \frac{(1-x)^2}{2} - \frac{(1-x)^2}{4} \, dx$$

$$= \frac{1}{4}\int_0^1 (1-x)^2 \, dx = \frac{1}{4}\int_0^1 1 - 2x + x^2 \, dx = \frac{1}{4}\Big| x - x^2 + \tfrac{1}{3}x^3 \Big|_0^1$$

$$= \frac{1}{4}\left(1 - 1 + \tfrac{1}{3}\right) = \underline{\underline{\tfrac{1}{12}}}$$