

## MATH 5399-001 MIDTERM EXAM DUE 15 APRIL

Before Noon on 15 April turn in your Macaulay 2 scripts and your handwritten or (much preferred) T<sub>E</sub>X'ed solutions to the following problems.

- (1) Given the  $f$ -vector of a simplicial complex  $\Delta$ , compute the dimension of  $k[\Delta]_i$ . [In high degrees you already have the formula, namely the Hilbert Polynomial.] Write a Macaulay 2 script that performs the computation; it shall take as input the  $f$ -vector and the degree  $i$ , and it shall return the dimension of  $k[\Delta]_i$ .
- (2) Let  $V = \{v_1, \dots, v_m\}$  be a set and let  $D \neq \emptyset$  be a subset of the power set of  $V$ .
  - (a) Show that  $D$  can be supplemented to a simplicial complex on a subset of  $V$ .
  - (b) Show that this can be done minimally. I.e. there is a minimal simplicial complex  $\Delta$  on a subset of  $V$  such that  $D \subset \Delta$ .
  - (c) Write a Macaulay 2 script that takes as input a subset of the power set of some set  $V$  and returns the Stanley-Reisner ring of the minimal simplicial complex found above.