

## MATH 5399-001 HOMEWORK DUE 14 MAR

Before 5 pm on 14 March turn in your handwritten or (much preferred)  $\text{\TeX}$ 'ed solutions to the following problems.

- (1) Rewrite each of the following polynomials, ordering the terms using the lex order, the grlex order, and the grevlex order.
  - (a)  $f(x, y, z) = 2x + 3y + z + x^2 - z^2 + x^3 - y^4$ .
  - (b)  $f(x, y, z) = 2x^2y^9 - 3x^5yz^4 + xyz^3 - xy^4$ .
- (2) In  $\mathbb{N}$  with the usual ordering, there are finitely many numbers between any two given numbers. Is this necessarily true for a monomial order on  $\mathbb{N}^n$ ?
- (3) Let  $A = [a_{ij}]$  be an  $m \times n$  matrix with real entries on row echelon form. Let  $J \subset \mathbb{R}[x_1, \dots, x_n]$  be the ideal generated by the linear forms  $\sum_{j=1}^n a_{ij}x_j$  for  $1 \leq i \leq m$ . Show that these generators form a Grobner basis for  $J$  with respect to a suitable monomial order.

Before Midnight on 14 March send me a Macaulay2 file (extension m2) with the commands to solve the following problems. Please make `restart` the first command in your file.

- (1) Modify the function from the previous homework to take a monomial order as a third argument and list the monomials in that order.