LIVING WITH A CONCEPT: PROOF BY INDUCTION

MATH 3310 SPRING 2019. PROJECT D.

Instructions: Over the course of four days, a new question related to proofs by induction will appear on Blackboard every day; your solutions are due at 11:59 pm on Monday–Thursday.

How would someone prove that for all nonnegative integers n, the number $6^n - 1$ is divisible by 5? Would it work to just check to see if the statement is true for $n = 0, 1, 2, 3, 4, 5$? Surely if a pattern is noticed for the first few cases, it should work for all cases, or?

Day 1. Consider the inequality $n^2 \leq 10000n$. Assume the goal is to prove that inequality is true for all positive integers n. A common mistake is to think that checking the inequality for numerous cases is enough to prove that statement is true in every case. First, verify that the inequality holds for $n = 1, 2, \cdots, 10$. Next, analyze the inequality; is there a positive integer n such that the inequality $n^2 \leq 10000n$ is not true?

Day 2. Every proof by induction needs a foundation to work off of, that is, something to get the induction started. We call this foundation the **Induction Base**. The base case involves verifying that the statement we want to prove is true for the first, or "least", case. For example, the base case for this week's proof, showing that $6^n - 1$ is divisible by 5 for all nonnegative integers n, requires showing that $6^n - 1$ is divisible by 5 when $n = 0$, since 0 is the first nonnegative integer. For tonight's task, simply verify that $6^n - 1$ is divisible by 5 when $n = 0$.

The domino analogy. One way to mentally picture how a proof by induction works is to think of the individual statements $p(n)$, $n \in \mathbb{N}$, as domino pieces placed upright in an infinite row. The induction base amounts to saying that you can make domino number 1 fall. In the Induction Hypothesis you assume that domino piece number k falls, and in the Induction Step you prove that it follows that also piece number $k+1$ falls. Now you know that piece number 1 falls, and you know that whenever some piece, call it number k, falls, then so does the next piece, number $k+1$. It follows that they all fall.

Day 3. In every proof, some assumptions are required. After proving the base case is true, an **Induction Hypothesis** needs to be made. The induction hypothesis is the assumption that the statement we want to prove is true for some number k. This might seem confusing, but think of it as assuming that the statement is true for some unspecified random case, which is called case k. For tonight's task, write the induction hypothesis for the proof that $6^n - 1$ is divisible by 5 for all nonnegative integers n