LIVING WITH A CONCEPT: PROOF BY CONTRADICTION

MATH 3310 SPRING 2019. PROJECT C.

Instructions: Over the course of four days, a new question related to partitions will appear on Blackboard every day; your solutions are due at 11:59 pm on Monday–Thursday.

> *Typically, a claim is proved by arguing why it has to be true. This week is dedicated to practicing how to prove a claim is true by showing that it cannot possibly be false.*

Day 1. Imagine that you are asked to prove the claim that there are no positive integer solutions to the equation $x^2 - y^2 = 1$. Restate the claim as a conditional statement. Would a direct proof be appropriate in this situation? Explain why or why not?

Day 2. A **proof by contradiction** differs from a direct proof. In a direct proof of a conditional statement, you assume the hypothesis to be true, and show it forces the conclusion to be true. In a proof by contradiction, the proof writer asks if it is possible for the hypothesis to be true and the conclusion to be false simultaneously? The writer assumes that the two happen simultaneously and works towards demonstrating that it contradicts some already established truth. Tonight's task is to prove the following statement:

Let a and b be integers. If a and b are odd, then $4 \nmid (a^2 + b^2)$.

Assume a and b are odd. Is it possible for $4|(a^2 + b^2)$? Would it contradict our assumption?

Day 3. On day 1, the following claim was mentioned: There are no positive integer solutions to the equation $x^2 - y^2 = 1$. Assume there are positive integer solutions. Explain why that cannot happen.

Day 4. Recall that a rational number is of the form $\frac{a}{b}$ where $a, b \in \mathbb{Z}$, $b \neq 0$. A typical example of a proof by contradiction is the one showing that $\sqrt{5}$ is irrational. Start off by assuming that $\sqrt{5}$ is rational, meaning $\sqrt{5} = \frac{a}{b}$, where the greatest common factor of a and b is 1. Then arithmetically manipulate the equation to show that it contradicts some aspect of that assumption.