LIVING WITH A CONCEPT: PARTITIONS

MATH 3310 SPRING 2019. PROJECT A.

Instructions: Over the course of four days, a new question related to partitions will appear on Blackboard every day; your solutions are due at 11.59 pm on Monday–Thursday.

Can we partition the set of integers $\mathbb{Z}$ into fifty infinite sets? We will work towards answering that question.

Day 1. First, we re-familiarize ourselves with the definition. Assume that you have a non-empty set A; define what a partition of A is.

Here we have a set

$$A = \{a, b, c, d, e, f, g\} .$$

Which of the following collections of subsets are partitions of A? For those that are not, explain which of the three conditions that is not satisfied.

$$\mathcal{S}_1 = \{\{a, e, f\}, \{b, c\}\}$$
$$\mathcal{S}_2 = \{\{a, b, c\}, \{c, d, e\}, \{a, f, g\}\}$$
$$\mathcal{S}_3 = \{\{a, e, g\}, \{b\}, \{c, d, f\}\}$$
$$\mathcal{S}_4 = \{\{a, b, c, d\}, \{\}, \{d, e, f, g\}\}$$

Day 2. Consider the set

$$X = \{1, 2, 3, 4, 5, 6, 7, 8, 9, 10\}.$$

If possible, partition it into the following number of subsets:

(a) 2 subsets
(b) 4 subsets
(c) 10 subsets
(d 12 subsets

Day 3. The sets

$$A = \{2n : n \in \mathbb{Z}\} \quad \text{and} \quad B = \{2n + 1 : n \in \mathbb{Z}\}$$

form a partition of $\mathbb{Z}$ where each set is infinite. You might be familiar with this partition. The elements in A are the even numbers and all the elements in B are the odd numbers. Can we find a different way to describe this partition? *Hint: Think of division with remainder.*

Day 4. Now that you see how to partition $\mathbb{Z}$ into 2 infinite subsets by using the different remainders one can obtain when dividing an integer by 2. Let that guide you as you complete the following tasks:

(a) How many different remainders can one get when dividing an integer by 3? List them.
(b) Every integer falls into one of three subsets depending on what the remainder is after division by 3. Using set builder notation, partition $\mathbb{Z}$ into 3 infinite subsets.
(c) Partition $\mathbb{Z}$ into 7 infinite subsets.