MATH 3310 HOMEWORK ASSIGNMENT 12

DUE ON FRIDAY 3 MAY 2019

(1) Find all integers that solve the congruence equation
$$x^2 + 2x \equiv_{12} 0\,.$$

(2) For eeach of the next functions $f\colon \mathbb{R} \to \mathbb{R}$ decide if it is 1-to-1 and/or onto. You must justify your answers.
  (a) $f(x) = x^3 - 2x + 7$.
  (b) $f(x) = e^x + x$.
  (c) $f(x) = 3x - e^x$.

(3) Consider the function $f\colon \mathbb{R} - \{1\} \to \mathbb{R}$ given by
$$f(x) = \frac{1}{x-1}\,.$$
  (a) Prove that f is injective.
  (b) Determine the range of f.
  (c) The corestriction $f\colon \mathbb{R} - \{1\} \to \text{range}(f)$ has an inverse, find it.

(4) Let A, B, and C be sets and consider functions $f\colon A \to B$ and $g\colon B \to C$. Prove that if the composite $g \circ f\colon A \to C$ is onto, then g is onto.

(5) Decide which of the statements (a)–(d) below that are proved by the following argument. Justify your answers.

> *Let A be a proper subset of B, that is $A \subseteq B$ and $A \neq B$, and assume that there exists a bijective function*
> $$f\colon A \to B\,.$$
> *Suppose B is finite, then A is finite as well, and since A is a proper subset of B one has $|A| < |B|$. The function f is 1-to-1, so one has $|f(A)| = |A|$. On the other hand, f is also onto, so one has $|f(A)| = |B|$. Contradiction!*

  (a) A set B cannot have the same cardinality as a proper subset A of B.
  (b) If a set B has the same cardinality as a proper subset A of B, then B is not finite.
  (c) The cardinality of a proper subset A of a set B is less than the cardinality of B.
  (d) If A is a proper subset of B, then there cannot exist a bijective function $f\colon A \to B$.