MATH 3310 HOMEWORK ASSIGNMENT 10

DUE ON FRIDAY 12 APRIL 2019

(1) Give a proof of the following statement:

The inequality $4^n > n^3$ *holds for all* $n \in \mathbb{N}$.

(2) Give a proof of the following statement:

Every integer in the recursively defined sequence below is odd.

$$x_1 = 1\,, \quad x_2 = 5\,, \quad \textit{and} \quad x_n = x_{n-1} + 4x_{n-2} \;\; \textit{for } n \geqslant 3$$

(3) Recall that the Fibonacci numbers are given by

$$F_1 = 1 = F_2 \quad \text{and} \quad F_n = F_{n-1} + F_{n-2} \text{ for } n \geqslant 3.$$

The number $s = \frac{1}{2}(1+\sqrt{5})$ is a solution to the equation $x^2 - x - 1 = 0$. Use this information to prove that $F_n \leqslant s^{n-1}$ holds for all $n \in \mathbb{N}$.

(4) Let R be the condition on $\mathbb{N}$ given by $a\ R\ b$ if $a|b$. Determine the set

$$\{b \in \mathbb{N} \mid b\ R^{-1}\ 4\}\,.$$

(5) For each of the following relations on $\{a, b, c, d, e\}$, decide which, if any, of the properties *reflexive*, *symmetric*, or *transitive* it has.

(a) $\{(a, a), (b, b), (c, c)\}$.
(b) $\{(a, a), (b, b), (c, c), (c, d), (d, c)\}$.
(c) $\{(a, b), (b, c), (c, d), (d, e), (d, a)\}$.
(d) $\{(a, b), (b, a), (c, b), (b, c), (a, c), (c, a)\}$.
(e) $\{(a, a), (b, b), (c, c), (c, d), (d, d), (e, e), (a, b), (e, a)\}$.
(f) $\{(a, a), (b, b), (c, c), (c, d), (d, d), (e, e), (e, a)\}$.