MATH 3310 HOMEWORK ASSIGNMENT 7

DUE ON FRIDAY 22 MARCH 2019

(1) For each of the following statements, provide a proof or a counterexample.
  (a) For every irrational number r there exists an irrational number s such that $r + s$ is rational.
  (b) For every $x \in \mathbb{R}$ one has $\sqrt{x^2} = x$.
  (c) There exists a rational number q such that for every irrational number r the product qr is rational.
  (d) For every real number $x \geqslant 0$ one has $x < x^2$.

(2) Prove that no upper bound for the interval $A = [0, 1)$ belongs to A.

(3) Consider the following argument:

Set $n = 2k + 1$. One then has

$$\begin{aligned} n(n+1) &= (2k+1)((2k+1)+1) \\ &= (2k+1)(2k+2) \\ &= 2(2k+1)(k+1)\,. \end{aligned}$$

As $(2k + 1)(k + 1)$ is an integer, it follows that $n(n + 1)$ is even.

For each of the next statements, decide if it is proved by the argument and explain why/why not?
  (a) For every integer n, the number $n(n + 1)$ is even.
  (b) If $n(n + 1)$ is odd, then so is n.
  (c) If n is odd, then $n(n + 1)$ is even.
  (d) If n is even, then $n(n + 1)$ is even.

(4) Prove the following statement:

For every $n \in \mathbb{Z}$ one has $3 | (n^2 + 2)$ if and only if $3 \nmid n$.

(5) Prove that the equation

$$x^4 + x^2 - 30 = 0$$

has a unique solution in the interval $(2, 3)$.