MATH 3310 HOMEWORK ASSIGNMENT 6

DUE ON FRIDAY 8 MARCH 2019

(1) Prove that an integer n is odd if and only if n^2 is odd.

(2) Prove the following statement.
Let $m, n \in \mathbb{Z}$. The product mn is odd if and only if m and n are both odd.

(3) Consider the following argument:

> *Let n be even and write $n = 2k$ for some $k \in \mathbb{Z}$. One then has*
> $$n^2 - n = (2k)^2 - 2k = 2(2k^2 - k),$$
> *and as $2k^2 - k$ is an integer, $n^2 - n$ is even.*

For each of the following statements, decide if it is proved by the argument and explain why/why not?
(a) If $n^2 - n$ is odd, then so is n.
(b) If n is odd, then so is $n^2 - n$.
(c) For every integer n, the number $n^2 - n$ is even.
(d) If n is even, then $n^2 - n$ is even.

(4) Prove the following statement:

For every integer n the integer $n^2 + n + 7$ is odd.

(5) Negate each of the following statements.
(a) $\forall x \in \mathbb{R}, x^2 \geqslant 0$
(b) $\forall x \in \mathbb{R}, \forall y \in \mathbb{R}, |x + y| \leqslant |x| + |y|$.
(c) $\exists n \in \mathbb{Z}, n + n = n^n$.
(d) $\exists p \in \mathbb{R} - \mathbb{Q}, \exists q \in \mathbb{R} - \mathbb{Q}, p^q \in \mathbb{Q}$.