

## MATH 3310 HOMEWORK ASSIGNMENT 4

DUE ON FRIDAY 15 FEBRUARY 2019

- (1) Construct open sentences  $p(x)$  and  $q(x)$  over the domain  $S = \mathcal{P}(\{\triangle, *\})$  such that the all of the following hold:
- For some  $x \in S$  both statements  $p(x)$  and  $q(x)$  are true
  - For some  $x \in S$  both statements  $p(x)$  and  $q(x)$  are false.
  - For some  $x \in S$  the statement  $p(x)$  is true and  $q(x)$  is false.
  - For some  $x \in S$  the statement  $p(x)$  is false and  $q(x)$  is true.

- (2) Consider the sets

$$X = \{x \in \mathbb{R} \mid |x| = 2\} \quad \text{and} \quad Y = \{y \in \mathbb{R} \mid |y| \leq 4\}.$$

Draw the subset  $(X \times Y) \cup (Y \times X)$  of  $\mathbb{R} \times \mathbb{R}$ .

- (3) Given the sets

$$A = \{1, 2, 4, 7, 11\} \quad \text{and} \quad B = \{1, 3, 4, 9\}$$

determine the truth value of the following statements:

- (a)  $(|A| = 5) \wedge (|B| = 4)$ .
- (b)  $(|A - B| = |A \cap B|) \vee (|A \cup B| = 9)$ .
- (c)  $(A \subseteq B) \implies (A \cap B = \emptyset)$ .

- (4) Construct a truth table to decide if the statement

$$(p \implies q) \wedge \sim q \implies \sim p$$

is a tautology.

- (5) Set  $S = \{2, 3, 4, 5, 6, 7, 8, 9, 10, 11\}$  and consider the open sentences

$$p(x): x \text{ is prime, } x \in S \quad \text{and} \quad q(x): 2^x - 1 \text{ is prime, } x \in S.$$

- (a) Determine all values of the variable  $x \in S$  for which the statement  $p(x) \implies q(x)$  is true.
- (b) Determine all values of the variable  $x \in S$  for which the statement  $q(x) \implies p(x)$  is true.
- (c) Determine all values of the variable  $x \in S$  for which the statement  $p(x) \iff q(x)$  is true.